

## THIN AIRFOIL THEORY

1. The basic premise of the theory is that for an airfoil in a uniform flow  $V_\infty$ , the airfoil can be replaced by a vortex sheet along the chord line. The strength of the vortex sheet,  $\gamma(x)$  is determined by the condition that the camber line must also be a streamline. This leads to the following *singular integral equation*

$$\frac{1}{2\pi} \oint_0^c \frac{\gamma(x)dx}{x_0 - x} = V_\infty[\alpha - (\frac{dz}{dx})_0]. \quad (1)$$

2. It is convenient to introduce the variables  $\theta$  and  $\theta_0$

$$x = \frac{c}{2}(1 - \cos\theta), \quad (2)$$

$$x_0 = \frac{c}{2}(1 - \cos\theta_0). \quad (3)$$

Then, we assume the following expansion for the strength of the vortex sheet

$$\gamma(\theta) = 2V_\infty(A_0 \frac{1 + \cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n \sin n\theta), \quad (4)$$

where  $A_0, A_1, A_2, \dots$  are constants to be determined in terms of the angle of attack  $\alpha$  and the slope of the camber line  $dz/dx$ .

3. Substituting (4) into (1) and carrying out the integration, gives

$$\frac{dz}{dx_0} = (\alpha - A_0) + \sum_{n=1}^{\infty} A_n \cos n\theta. \quad (5)$$

Thus,  $(\alpha - A_0), A_1, A_2, \dots$  are the Fourier coefficients of  $dz/dx$ . Therefore, we have

$$A_0 = \alpha - \frac{1}{\pi} \int_0^\pi \frac{dz}{dx} d\theta, \quad (6)$$

$$A_n = \frac{2}{\pi} \int_0^\pi \frac{dz}{dx} \cos n\theta d\theta \quad \text{for } n = 1, 2, \dots \quad (7)$$

$$(8)$$

4. All aerodynamic quantities can now be calculated from  $A_0, A_1, A_2, \dots$

$$\Gamma = \pi c V_\infty (A_0 + \frac{A_1}{2}), \quad (9)$$

$$c_l = \pi(2A_0 + A_1), \quad (10)$$

$$c_{m,le} = -\frac{\pi}{2}(A_0 + A_1 - \frac{A_2}{2}) = -\frac{c_l}{4} + \frac{\pi}{4}(A_2 - A_1), \quad (11)$$

$$x_{cp} = \frac{c}{4}[1 + \frac{\pi}{c_l}(A_1 - A_2)], \quad (12)$$

$$c_{mac} = c_{m_{c/4}} = \frac{\pi}{4}(A_2 - A_1). \quad (13)$$