# Bandwidth Filters

Instruments used to analyze noise have either constant bandwidth or proportional bandwidth devices.

The constant bandwidth is essentially a tunable narrow band filter with constant bandwidth, $w = f_u - f_\ell$, where $f_u$ and $f_\ell$ are the upper and lower half-power frequencies. The center frequency of the filter, defined in general as

$$f_c = \sqrt{f_u f_\ell} \tag{1}$$

is usually variable so that the filter can be swept over the desired frequency range. Bandwidths range from a few tens of a hertz to less than a few hundredths of a hertz.

The proportional bandwidth instrument consists of a series of relatively broadband filters with upper and lower half-power frequencies satisfying the relationship $f_u/f_\ell$ = constant. Each bandwidth, being proportional to the center frequency, increases with increasing frequencies with contiguous bands. Common instruments of this type are the *octave-band* filter with $f_u/f_\ell = 2$, the *1/3-octave-band* filter with $f_u/f_\ell = 2^{1/3}$, and the *1/10-octave-band* filter with $f_u/f_\ell = 2^{1/10}$.

As an example, let us derive the *1/3-octave-band* filter width in terms of the center frequency.

$$w = (\sqrt{2^{\frac{1}{3}}} - \frac{1}{\sqrt{2^{\frac{1}{3}}}})f_c, \tag{2}$$

which gives

$$w = 0.232 f_c. \tag{3}$$

In many problems, we deal with the power spectral density normalized with respect to $\omega = 2\pi f_c$. In this case,

$$\Delta\omega = 0.232(2\pi)f_c = 0.232\omega_c. \tag{4}$$

In decibels, this implies that we add

$$10 log_{10}\omega_c - 6.353.$$

**Table 2-1 Center, lower, and upper frequencies for $\frac{1}{3}$-octave bands**

| Band no. | Frequency, Hz | | |
|---|---|---|---|
| | Center | Lower | Upper |
| 12 | 16† | 14.0 | 18.0 |
| 13 | 20 | 18.0 | 22.4† |
| 14 | 25 | 22.4† | 28.0 |
| 15 | 31.5† | 28.0 | 35.5 |
| 16 | 40 | 35.5 | 45† |
| 17 | 50 | 45† | 56 |
| 18 | 63† | 56 | 71 |
| 19 | 80 | 71 | 90† |
| 20 | 100 | 90† | 112 |
| 21 | 125† | 112 | 140 |
| 22 | 160 | 140 | 180† |
| 23 | 200 | 180† | 224 |
| 24 | 250† | 224 | 280 |
| 25 | 315 | 280 | 355† |
| 26 | 400 | 355† | 450 |
| 27 | 500† | 450 | 560 |
| 28 | 630 | 560 | 710† |
| 29 | 800 | 710† | 900 |
| 30 | 1,000† | 900 | 1,120 |
| 31 | 1,250 | 1,120 | 1,400† |
| 32 | 1,600 | 1,400† | 1,800 |
| 33 | 2,000† | 1,800 | 2,240 |
| 34 | 2,500 | 2,240 | 2,800† |
| 35 | 3,150 | 2,800† | 3,550 |
| 36 | 4,000† | 3,550 | 4,500 |
| 37 | 5,000 | 4,500 | 5,600† |
| 38 | 6,300 | 5,600† | 7,100 |
| 39 | 8,000† | 7,100 | 9,000 |
| 40 | 10,000 | 9,000 | 11,200† |
| 41 | 12,500 | 11,200† | 14,000 |
| 42 | 16,000† | 14,000 | 18,000 |
| 43 | 20,000 | 18,000 | 22,400† |
| 44 | 25,000 | 22,400† | 28,000 |
| 45 | 31,500† | 28,000 | 35,500 |