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Introduction to Acoustics

Homework 2

1. Two superimposed plane waves are propagating in the $+x$ and $-x$ directions such that the pressure is given by

$$p' = \mathbf{A}e^{-i\omega(t-x/c)} + \mathbf{B}e^{-i\omega(t+x/c)}, \tag{1}$$

where $\mathbf{A}$ and $\mathbf{B}$ are complex amplitudes defined by $\mathbf{A} = A\,exp(i\varphi)$ and $\mathbf{B} = B\,exp(i\psi)$. Calculate the average intensity $\bar{I}_x$ in the $+x$ direction. How does $\bar{I}_x$ vary with x?
Hint: Before traeting the general case,consider the various cases where (1) $\mathbf{A} = \mathbf{B}$, (2) $A = B$ but $\varphi \neq \psi$, (3) $A \neq B$ but $\varphi = \psi$ and finally (4) $A \neq B$ and $\varphi \neq \psi$.

2. The speed of sound c in distilled water depends on temperature and pressure. From measurements, it was found that

$$c(p,t) = 1402.7 + 488t - 482t^2 + 135t^3 + 10^{-7}(p - p_{atm})(15.9 + 2.8t + 2.4t^2), \tag{2}$$

where p is the pressure in Pascals (Pa) and $t = T/100$, with T in degree Celsius. This equation is accurate to within 0.05 percent for $0 \leq T \leq 100^o$C and $0 \leq p \leq 2 \times 10^7$ Pa. Plot the variation of the speed of sound $c(p,t)$ in water versus T as the temperature varies from 0^o C to 100^o C for $p = p_{atm}$ and $p = 10p_{atm}$. $p_{atm} = 1.013 \times 10^5$ Pa.

3. Find the intensity level in dB *re* $10^{-12} W/m^2$ of a plane wave propagating in air and having an effective acoustic pressure of $1\mu\,bar$.

4. Find the intensity level in W/m^2 produced by an acoustic plane wave in water of sound pressure level (SPL) ($re\ 1\mu\,bar$) $= 120\,dB$.

5. What is the ratio of the acoustic pressure in water for a plane wave to that of a similar wave in air of equal intensity?

6. Two harmonic plane waves with the same pressure amplitude but with different frequencies are traveling in the x direction:

$$p'_1 = \hat{p}e^{i\omega_1(t-x/c)}, \quad (3)$$
$$p'_2 = \hat{p}e^{i\omega_2(t-x/c)}. \quad (4)$$

1. Show that the resultant acoustic wave can be written as

   $$p' = p'_1 + p'_2 = 2\hat{p}cos[\Delta\omega(t - x/c)]e^{i\bar{\omega}(t-x/c)}, \quad (5)$$

   where $\bar{\omega} = (\omega_1 + \omega_2)/2$ and $\Delta\omega = (\omega_1 - \omega_2)/2$. Equation (5) shows that the resulting plane wave has a modulated wave-like amplitude

   $$A = 2\hat{p}cos[\Delta\omega(t - x/c)]. \quad (6)$$

   In what follows we assume $\Delta\omega << \bar{\omega}$.

2. What are the period T_A and wave length λ_A associated with the modulated amplitude A. If $\Delta\omega = 0.2\bar{\omega}$, plot A at $t = 0$ over two wavelengths $2\lambda_A$. Noting that A is the envelope for (5), plot p' at $t = 0$ over two wavelengths $2\lambda_A$. How many full waves the acoustic wave (5) has per wavelength λ_A?

3. Calculate the instantaneous energy density E_i and the average density defined by

   $$E = \frac{1}{\bar{T}}\int_0^{\bar{T}} E_i dt, \quad (7)$$

   where $\bar{T} = 2\pi/\bar{\omega}$ is the period associated with the phase of the resultant acoustic wave (5).

   Hint: if $\Delta\omega << \bar{\omega}$, A is almost constant over a period $\bar{T}$.