Aeroacoustic and Aerodynamics of Swirling Flows*

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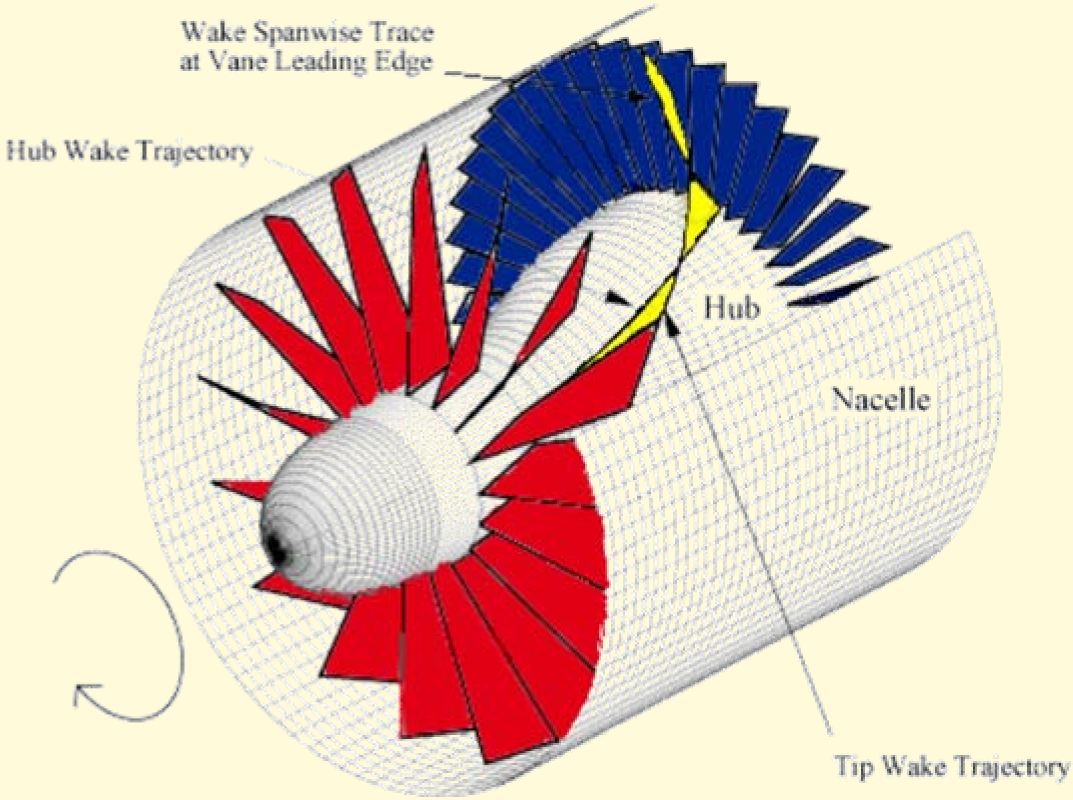
OVERVIEW OF PRESENTATION

- Disturbances in Swirling Flows
- Normal Mode Analysis
- Application to Computational Aeroacoustics
- Vortical Disturbances
- Aerodynamic and Acoustic Blade Response
- Conclusions

Swirling Flow in a Fan



Issues For Consideration

- Effect of swirl on aeroacoustics and aerodynamics?
- Can we consider separately acoustic, vortical and entropic disturbances?
- How does swirl affect sound propagation (trapped modes)?
- How do vortical disturbances propagate?
- How strong is the coupling between pressure, vortical and entropic modes?
- What are the conditions for flow instability?
- What are the boundary conditions to be specified?

Scaling Analysis

❖ Acoustic phenomena:

- Acoustic frequency: nB Ω
- Rossby number = $\frac{nB\,\Omega\, r_t}{c_0} >> 1$

❖ Convected Disturbances:

- Convection Frequency ~ Shaft Frequency Ω
- Rossby number = $\frac{\Omega r_t}{U_x} \approx O(1)$
- Wakes are distorted as they convect at different velocity. Centrifugal and Coriolis accelerations create force imbalance which modifies amplitude and phase and may cause hydrodynamic instability.

Mathematical Formulation

- Linearized Euler equations
- Axisymmetric swirling mean flow

$$\vec{U}(\vec{x}) = U_x(x,r)\vec{e}_x + U_s(x,r)\vec{e}_\theta$$

- Mean flow is obtained from data or computation
- For analysis the swirl velocity is taken

$$U_s = \Omega r + \frac{\Gamma}{r}$$

- The stagnation enthalpy, entropy, velocity and vorticity are related by Crocco's equation

$$\nabla H = T\nabla S + \mathbf{U} \times \zeta$$

Normal Mode Analysis

Normal Mode Analysis

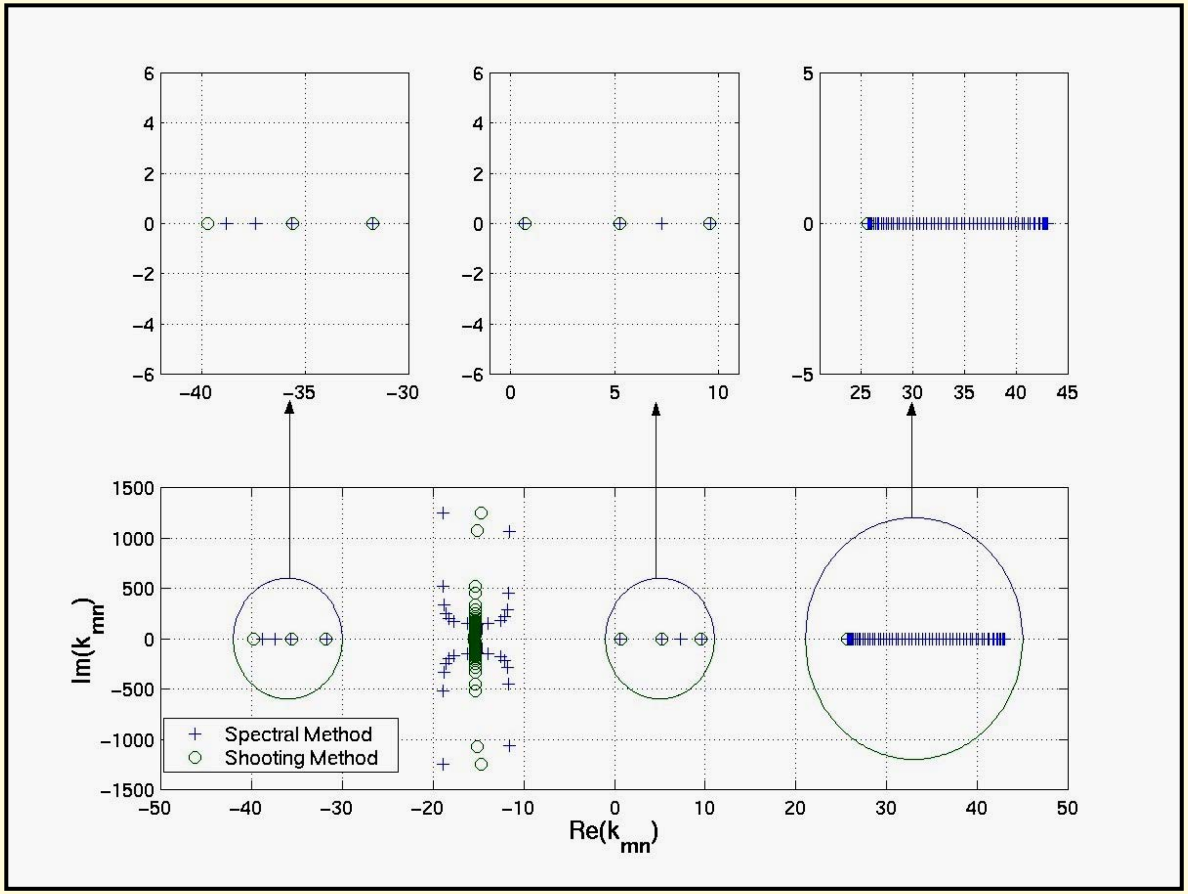
- A normal mode analysis of linearized Euler equations is carried out assuming solutions of the form

$$f(r)e^{i(-\omega\, t+m_\nu\theta+k_{mn}x)}$$

- Eigenvalue problem is not a Sturm-Liouville type
- A combination of spectral and shooting methods is used in solving this problem
    - Spectral method produces spurious acoustic modes
    - Shooting method is used to eliminate the spurious modes and to increase the accuracy of the acoustic modes

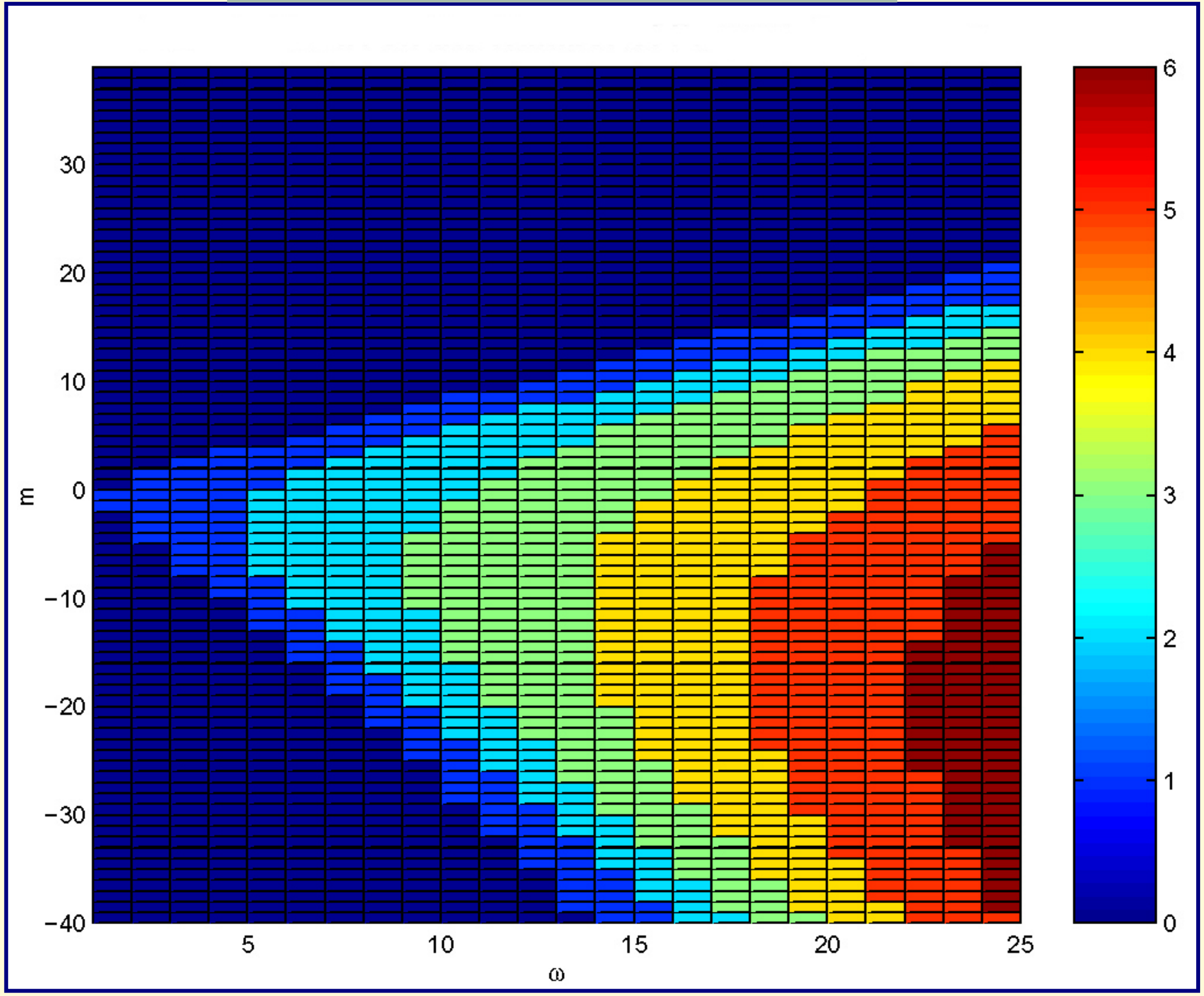
Comparison Between the Spectral and Shooting Methods

M_x=0.55, M_Γ=0.24, M_Ω=0.21, ω=16, and m=-1



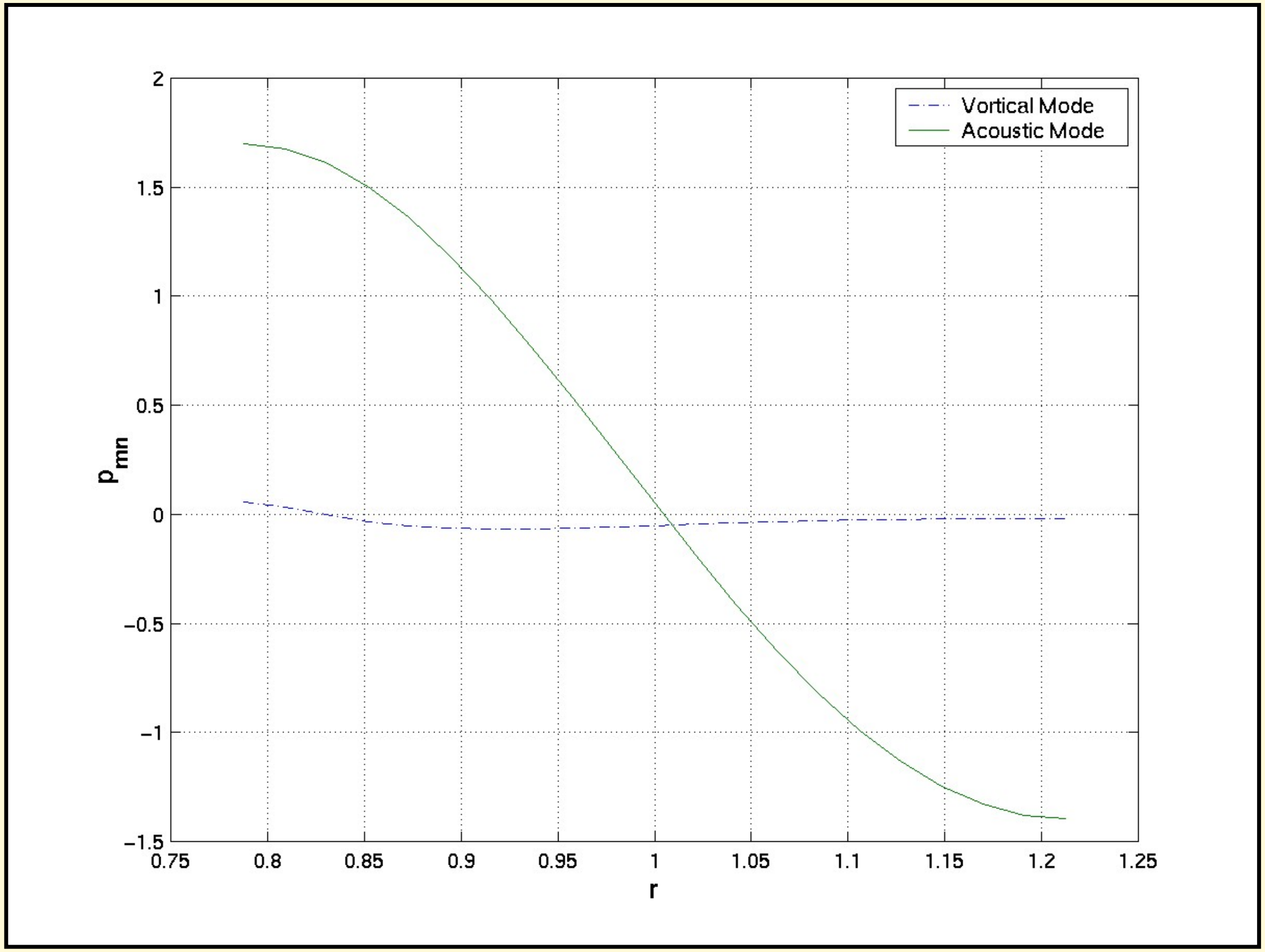
Effect of Swirl on Eigenmode Distribution

$M_{xm=}0.56$, $M_{\Gamma}=0.25$, $M_{\Omega}=0.21$

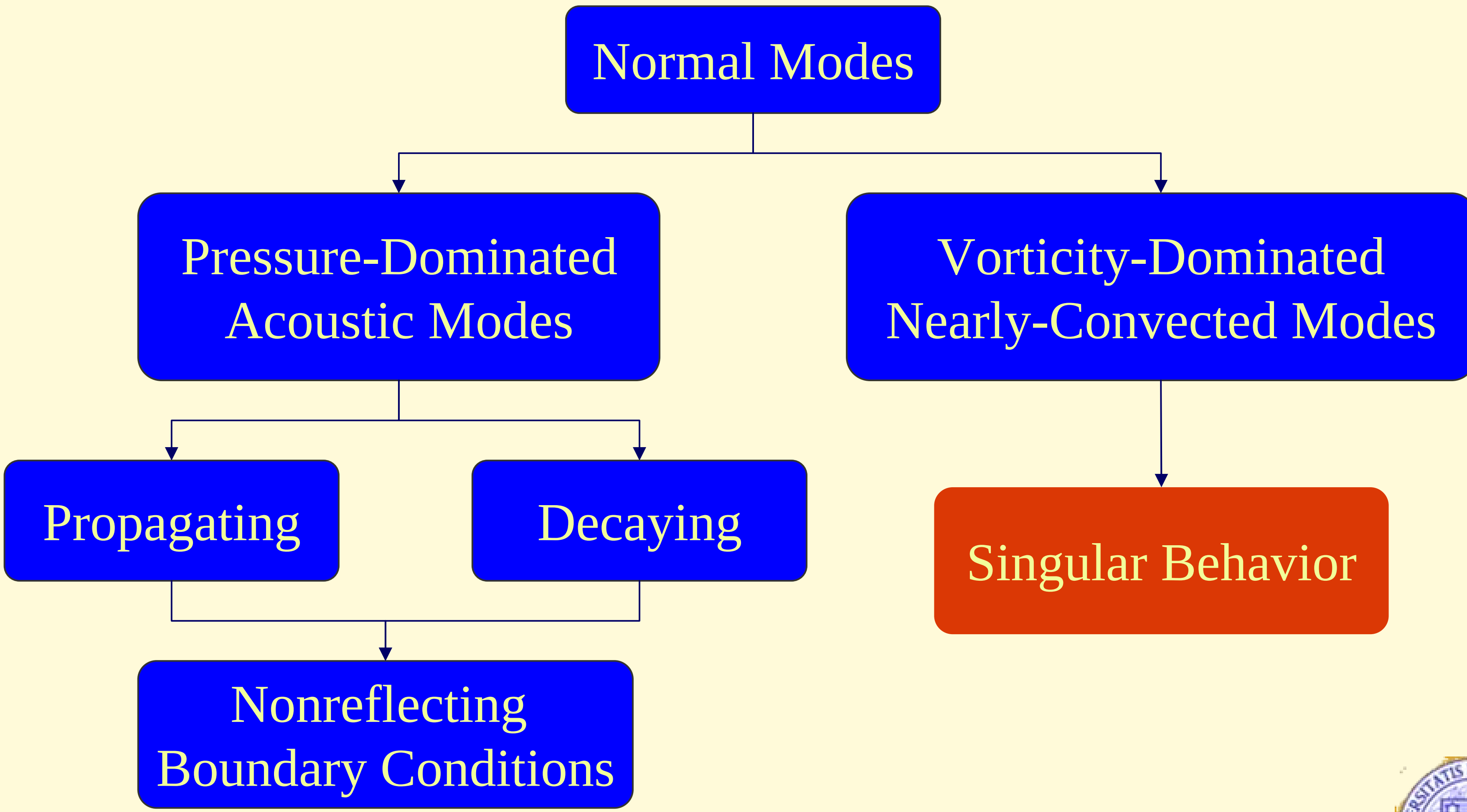


Pressure Content of Acoustic and Vortical Modes

M_x=0.5, M_Γ=0.2, M_Ω=0.2, ω=2π, and m=-1

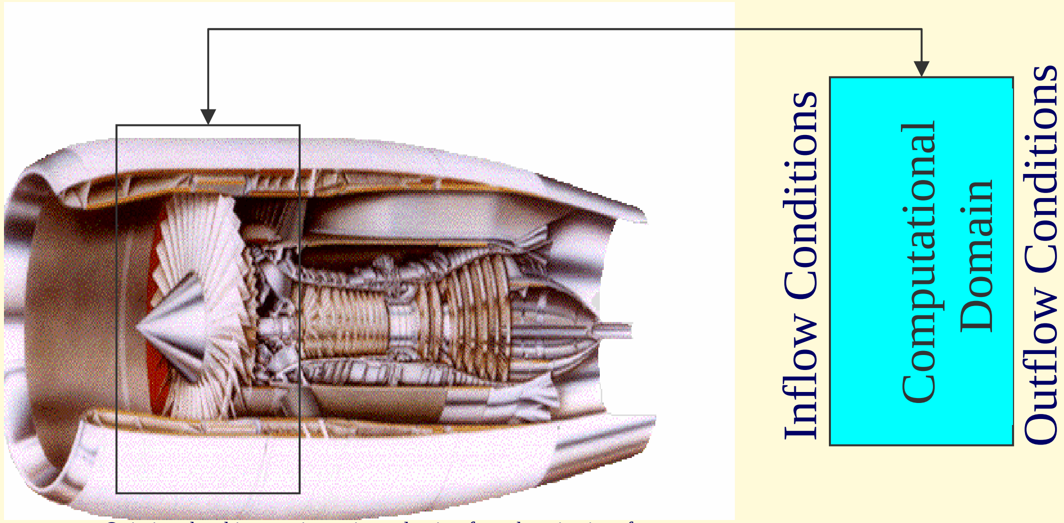


Summary of Normal Mode Analysis



Nonreflecting Boundary Conditions

- Accurate nonreflecting boundary conditions are necessary for computational aeroacoustics



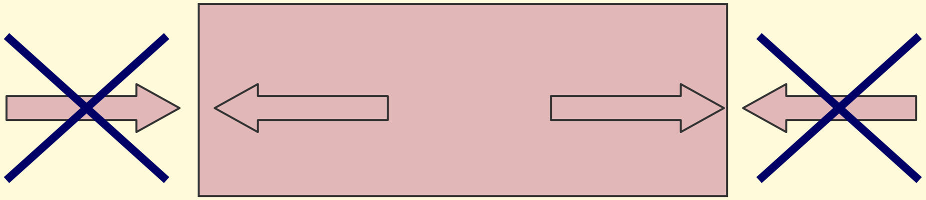
Quieting the skies: engine noise reduction for subsonic aircraft
Advanced subsonic technology program. NASA Lewis research center, Cleveland, Ohio

Formulation

- Presssure at the boundaries is expanded in terms of the acoustic eigenmodes.

$$p(\vec{x},t) = \int_{\omega} \sum_{\nu=-\infty}^{\infty} \sum_{n=0}^{\infty} c_{mn} p_{mn}(\omega, r) e^{i(-\omega t + m_{\nu}\theta + k_{mn}x)} d\omega$$

- Only outgoing modes are used in the expansion.
- Group velocity is used to determine outgoing modes.



Causality

Nonreflecting Boundary Conditions (Cont.)

$$p(\vec{x},t) = \sum_{\nu=-M/2}^{M/2} \sum_{n=0}^{N} c_{mn}\, p_{mn}(r)\, e^{i(-\omega t + m_\nu \theta + k_{mn} x)}$$

$$\left[p\right] = \left[\Re\right]\left[c\right]$$

$$\left[p\right]_{L-1} = \left[\Re\right]_{L-1}\left[c\right]$$

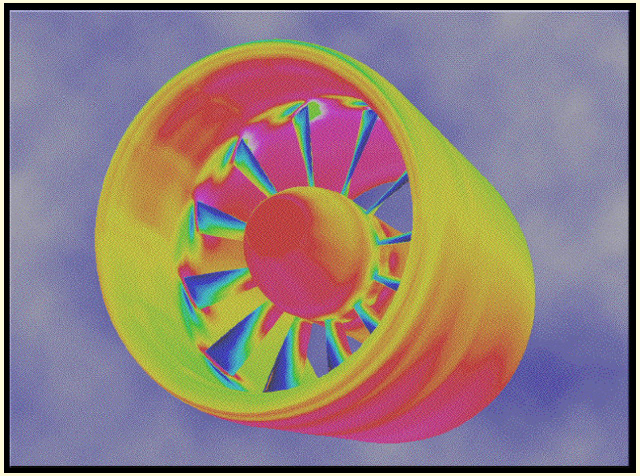
$$\left[p\right]_{L} = \left[\Re\right]_{L}\left[c\right]$$

$$\left[p\right]_{L} = \left[\Re\right]_{L}\left[\Re\right]_{L-1}^{-1}\left[p\right]_{L-1}$$

Application to Computational Aeroacoustics

Test Problems for Acoustic Waves

- **Acoustic waves and/or a combination of acoustic and vortical waves are imposed upstream of an annular duct with swirling mean flow and nonreflecting boundary condition applied downstream**



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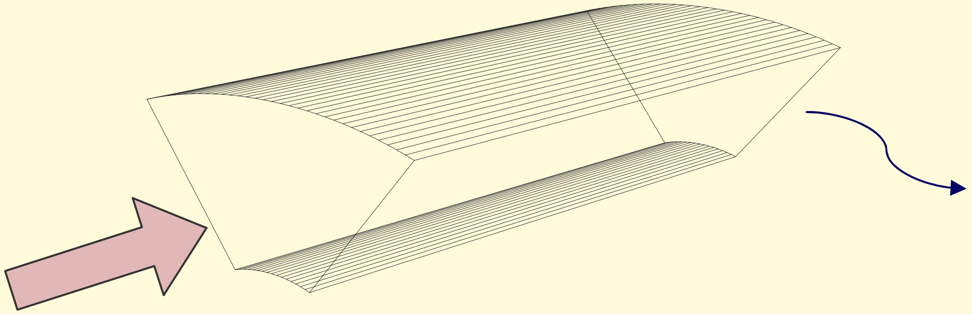
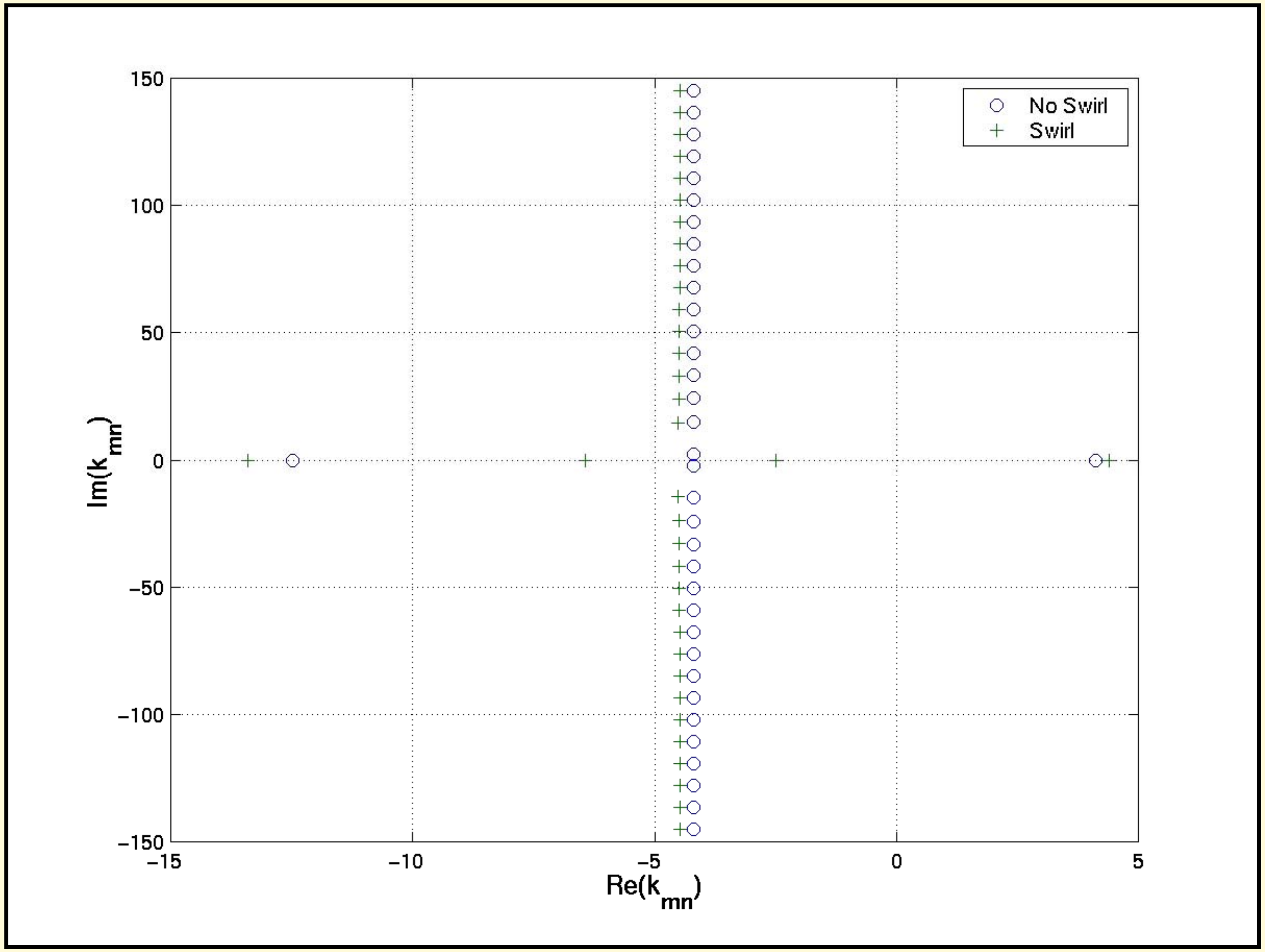
Acoustic and /or Vortical Mode



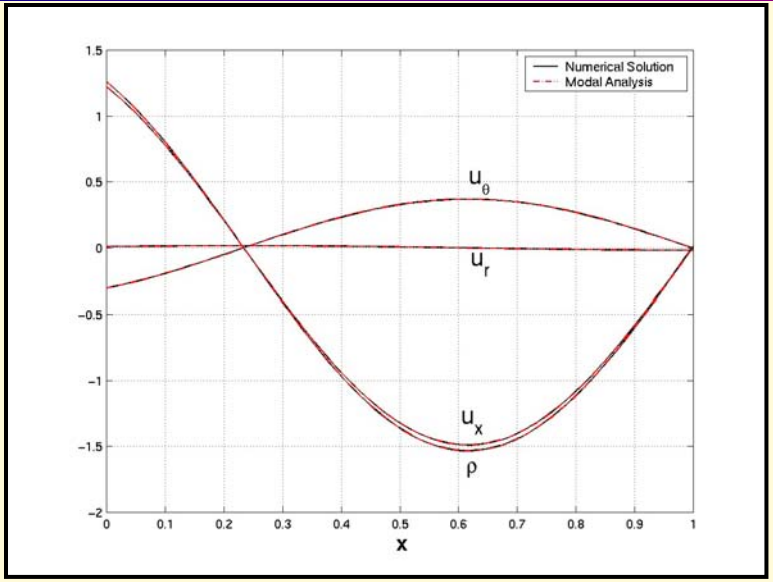
Nonreflecting Boundary conditions

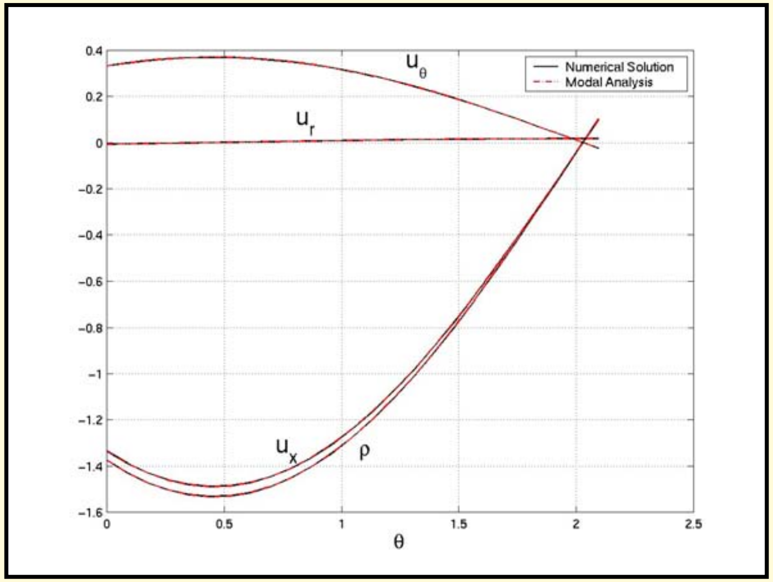
Acoustic Normal Mode Spectrum

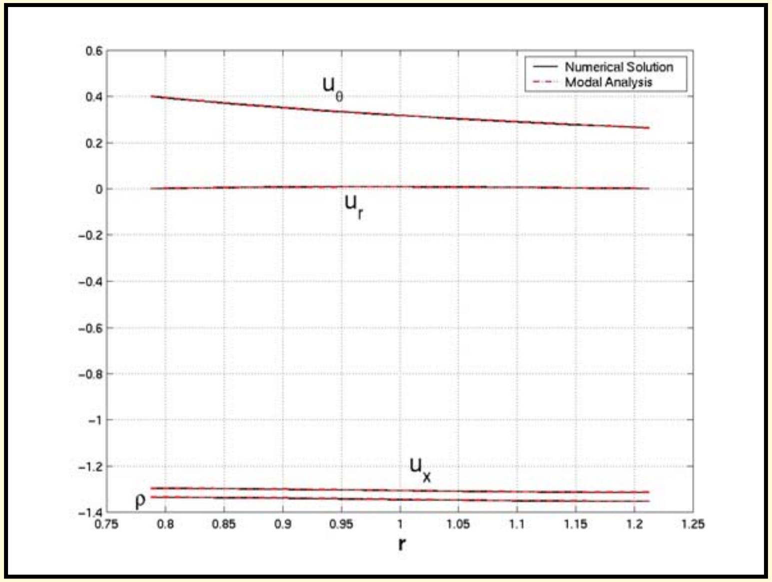
M_x=0.5, M_Γ=0.2, M_Ω=0.2, $\omega=2\pi$, and m=-1



Density and Velocity Distribution in Uniform Flow

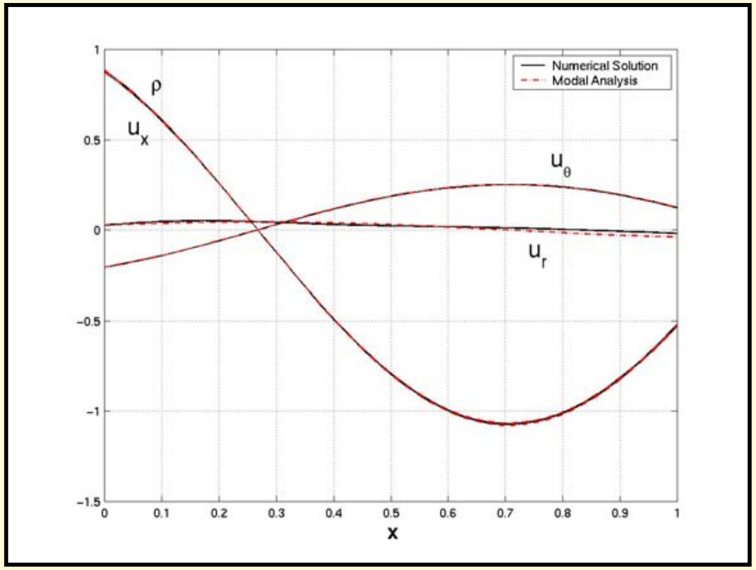


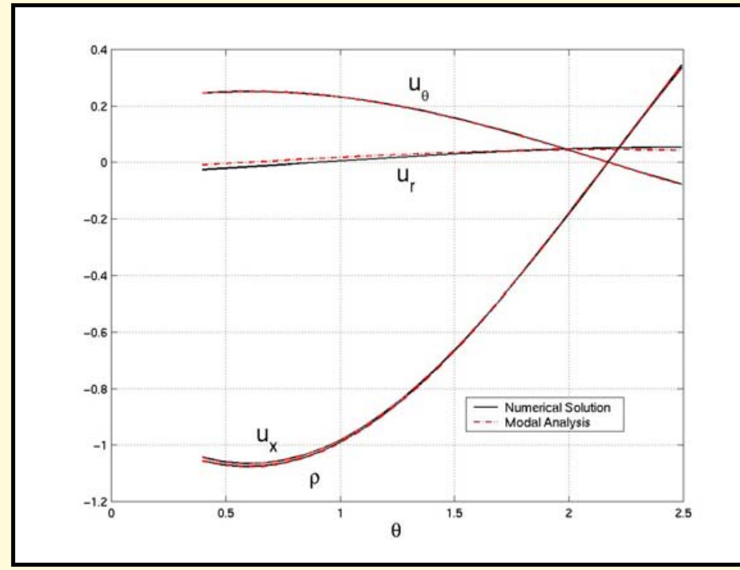


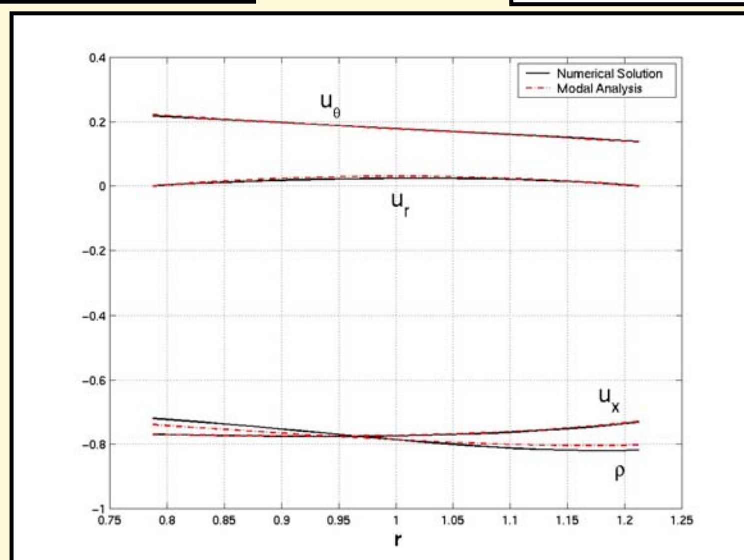


$$k_{-1,1} = 4.1077$$

Density and Velocity Distribution in Swirling Flow



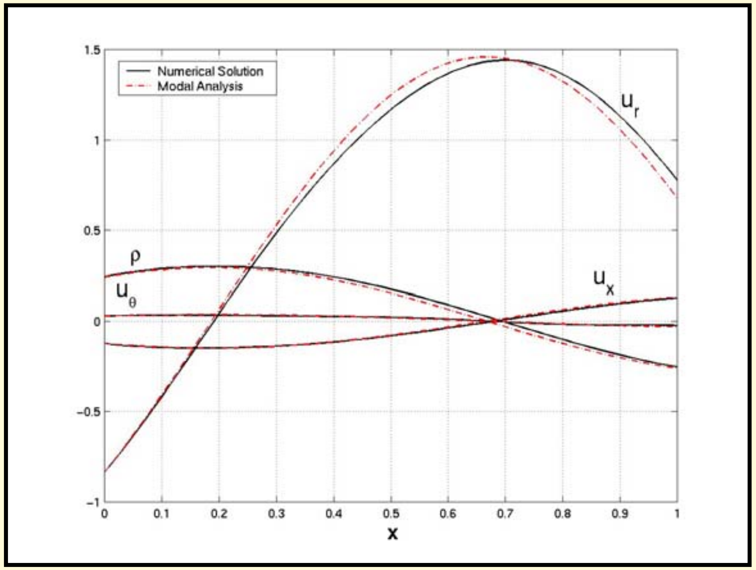


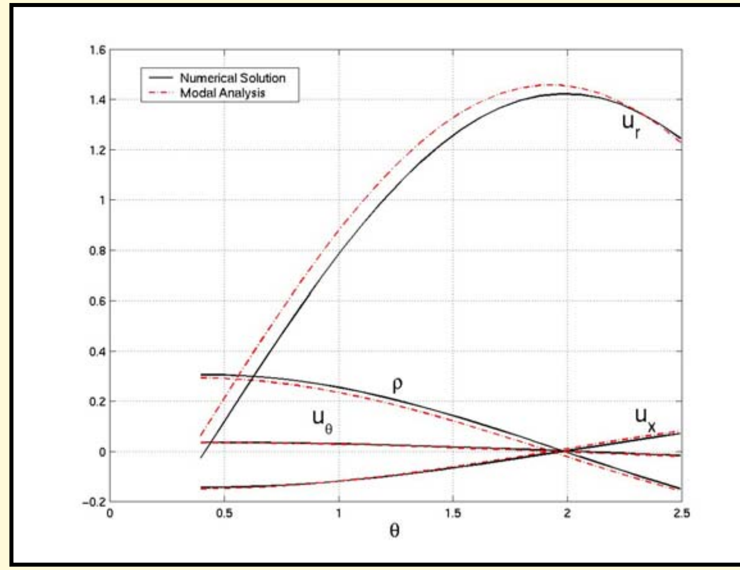


First Propagating Acoustic Mode

$$k_{-1,1} = 4.3942$$

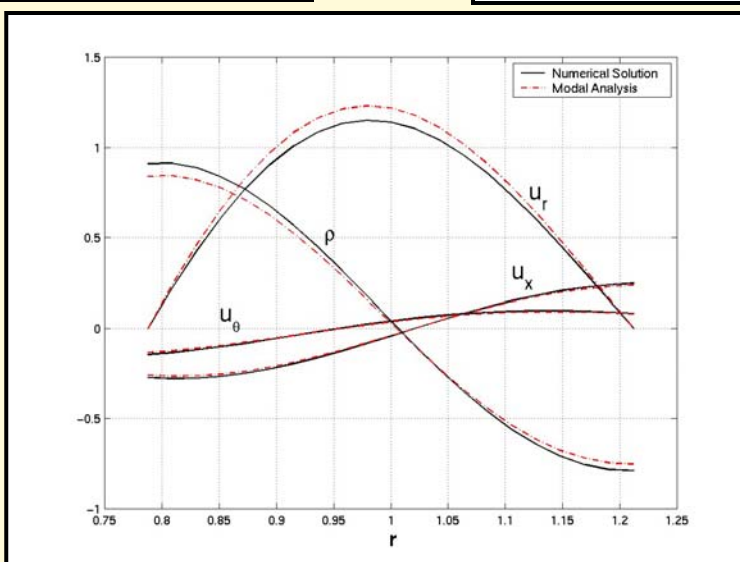
Density and Velocity Distribution in Swirling Flow



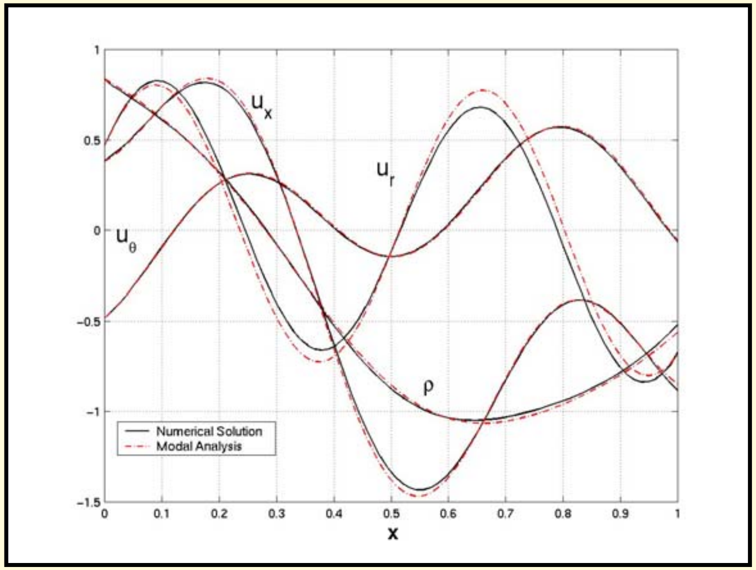


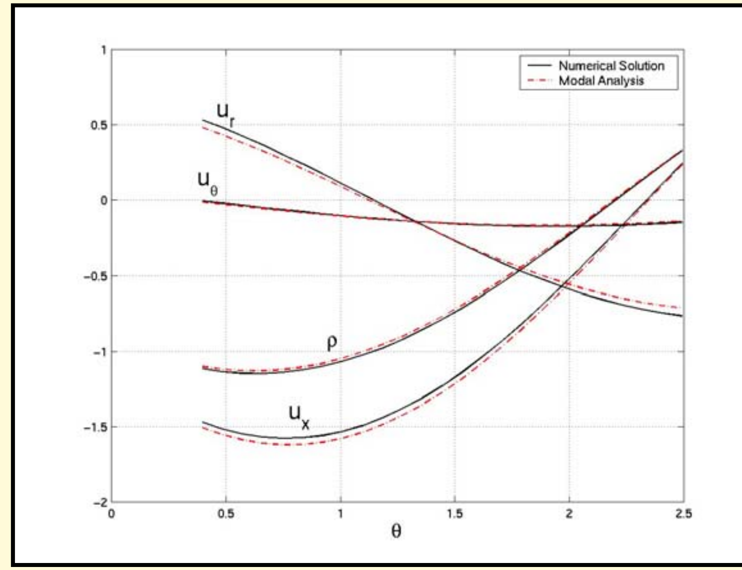
Second Propagating Acoustic Mode

$$k_{-1,2} = -2.4639$$



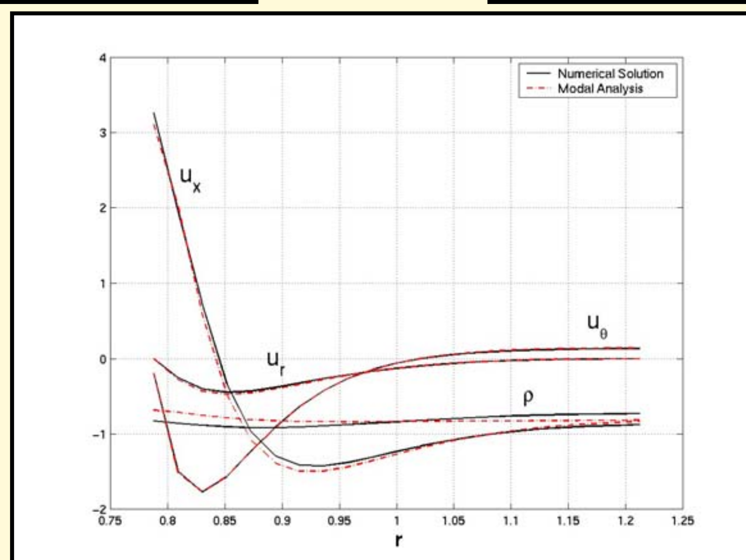
Density and Velocity Distribution in Swirling Flow





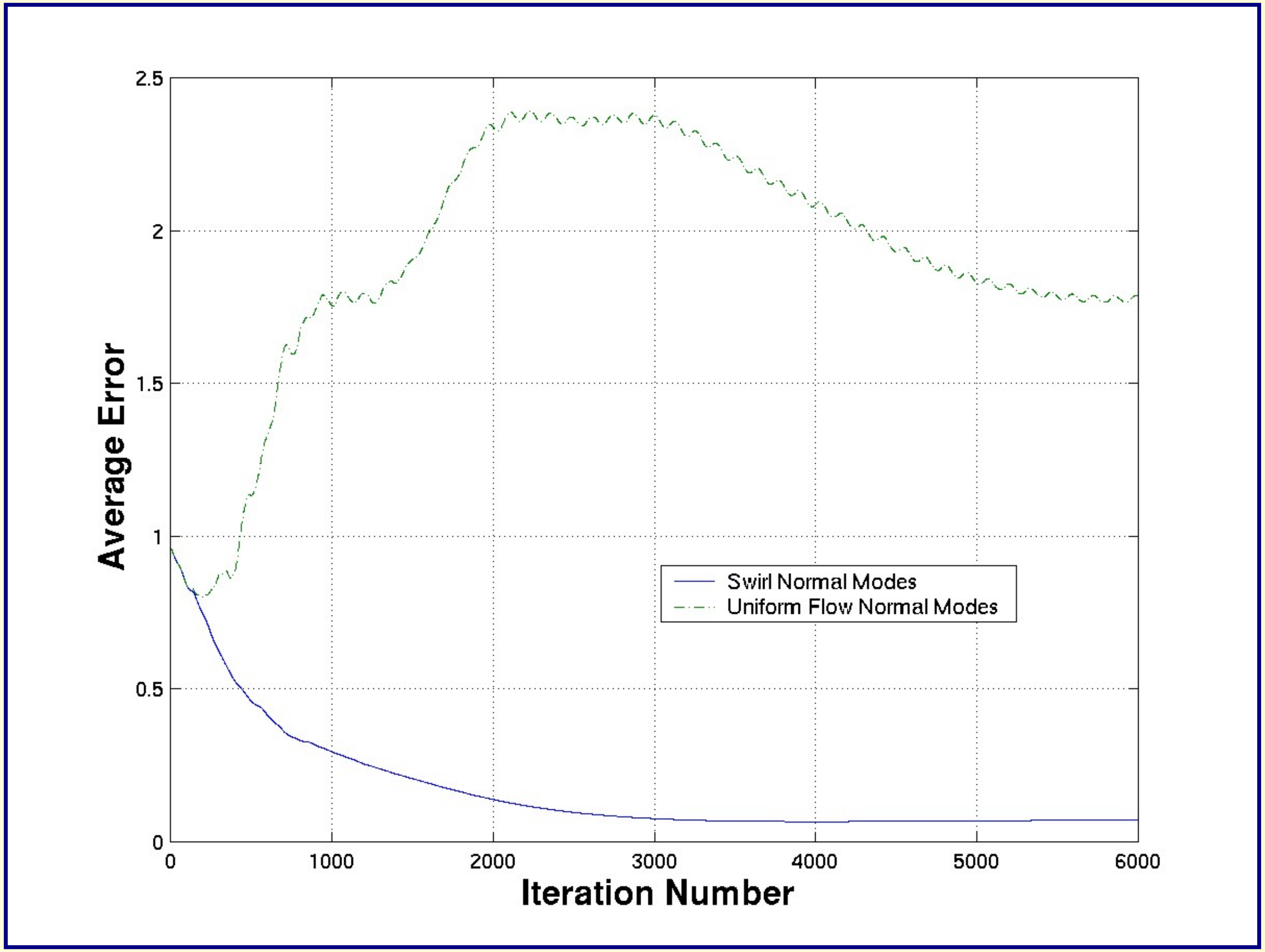
Acoustic & Vortical Modes

$$k_{-1,1} = 4.3942$$
$$k_{-1,3} = 11.7626$$



Sensitivity of Numerical Solutions to Accuracy of Eigenvalue

M_x=0.5, M_Γ=0.2, M_Ω=0.2, $\omega=2\pi$, and m=-1

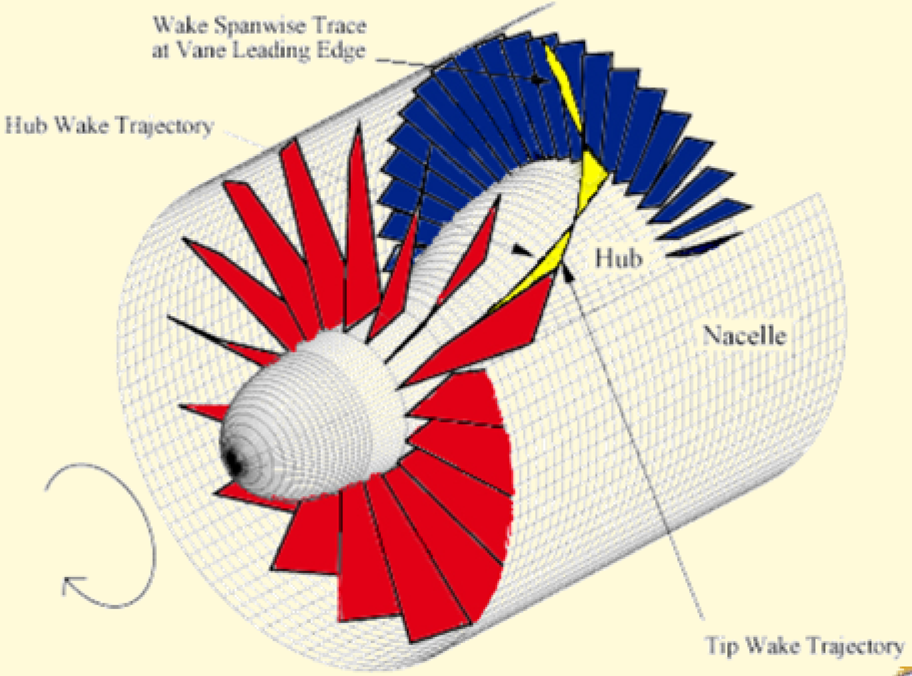


Vortical Disturbances

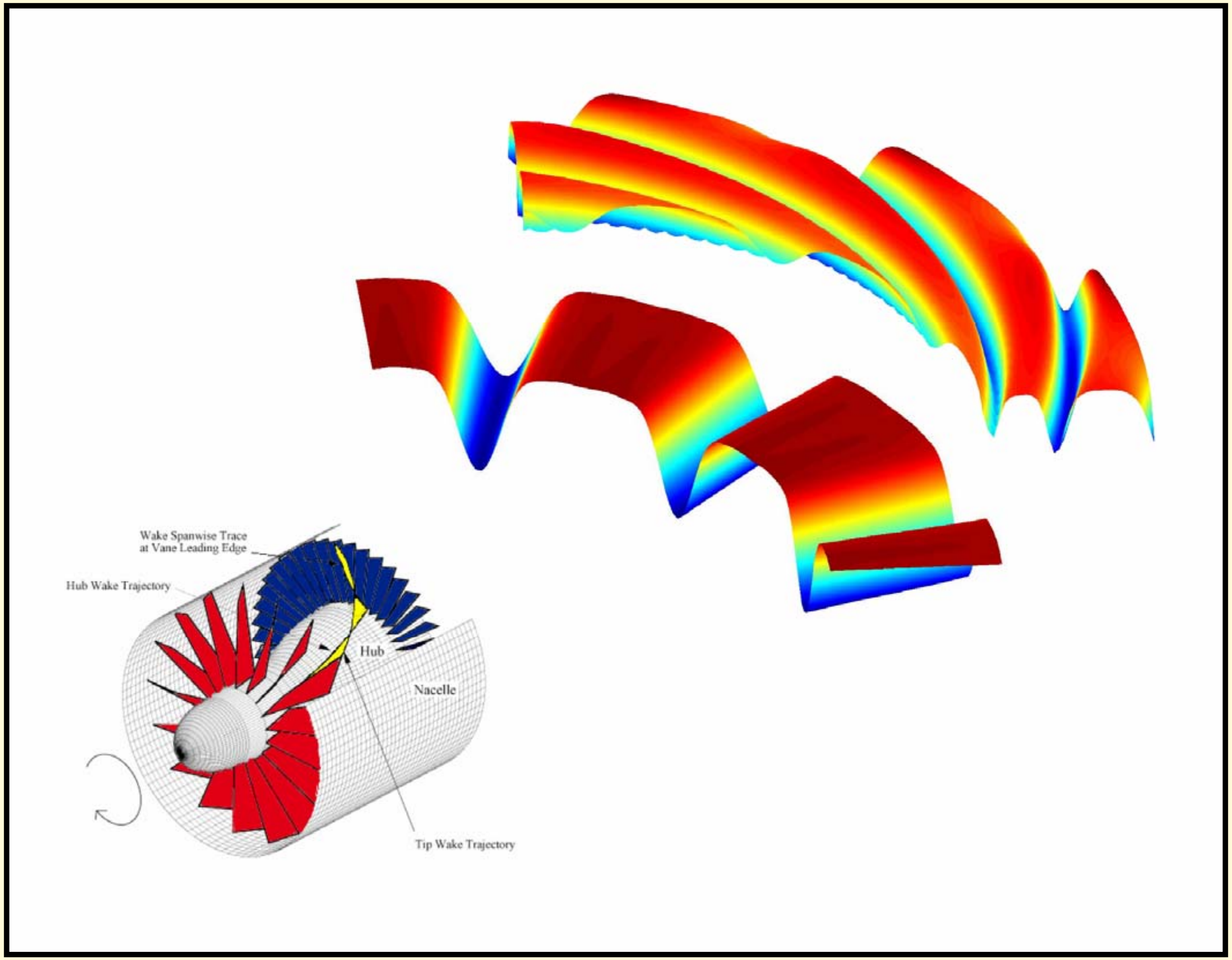
Initial Value Solution

$$u(x,r,\theta,t) = \int_{-\infty}^{+\infty} \sum_{m=-\infty}^{m=+\infty} A_m(x,r)e^{i(\alpha x + m\theta - \omega t)} d\omega$$

$$\frac{D_o}{Dt}(\alpha x + m\theta - \omega t) = 0$$



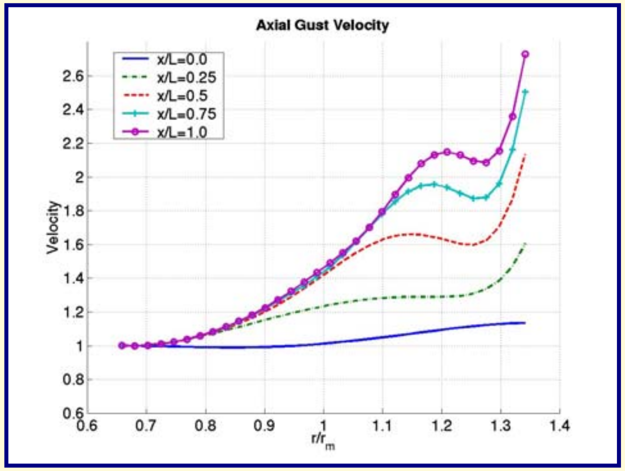
Wake Distortion by Swirl

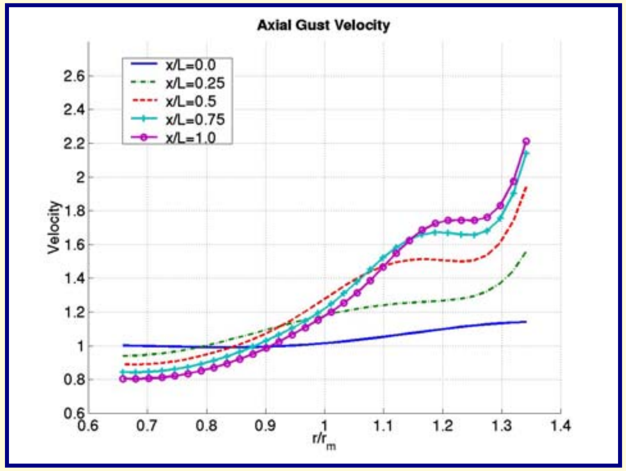


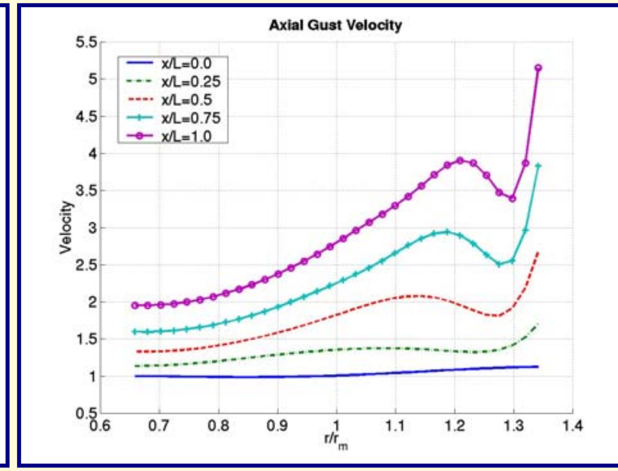
Accelerating axial flow

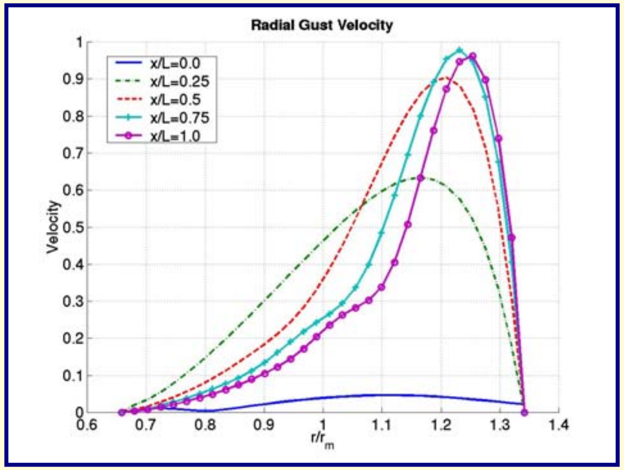
$$\mathbf{U}(x,r) = 85\left(1+\gamma\frac{x}{L}\right)\mathbf{e}_x + \frac{50}{r}\mathbf{e}_\theta,\ m = 10,\ \omega = 5000$$

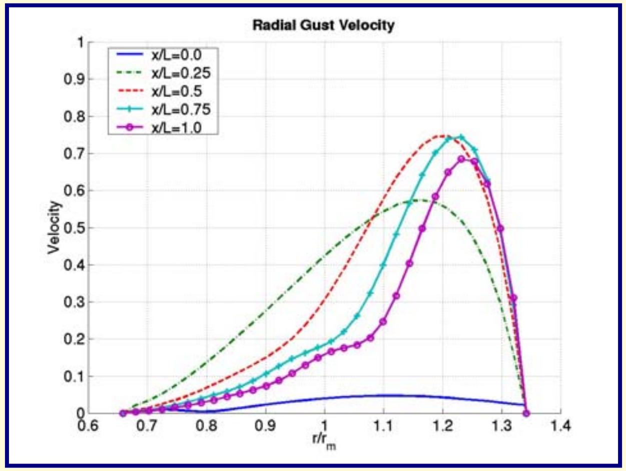
$\gamma = 0$	$\gamma > 0$	$\gamma < 0$

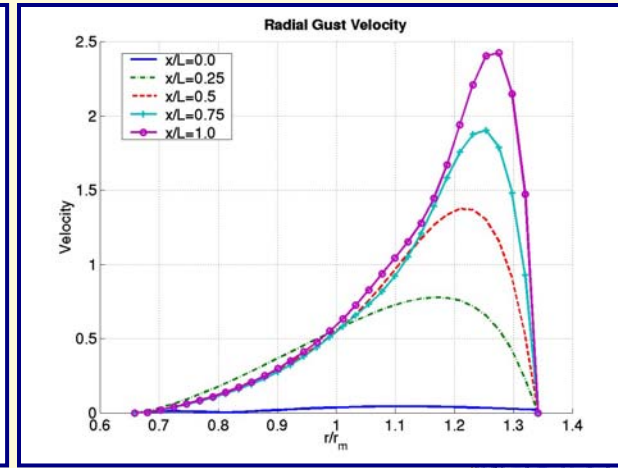










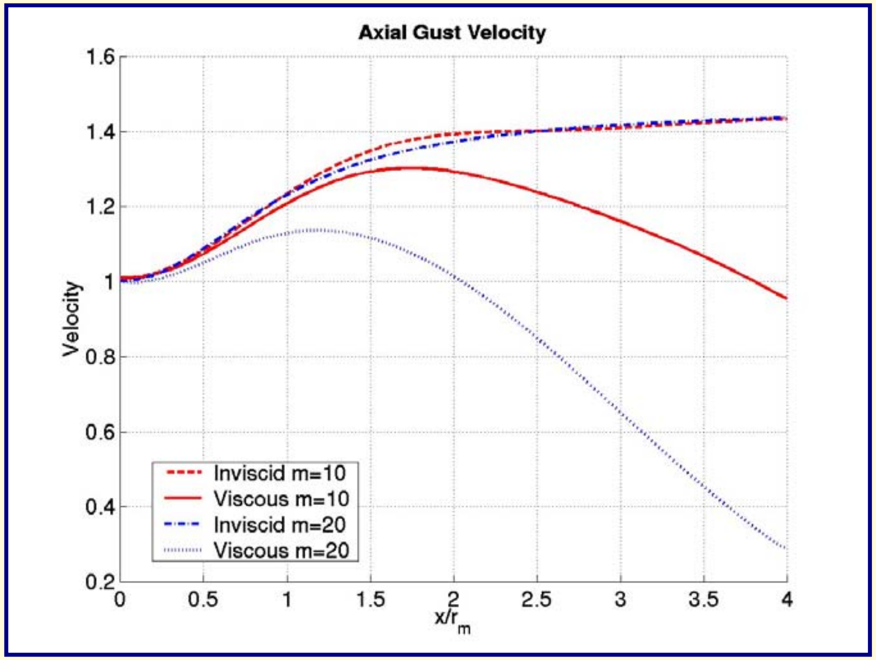


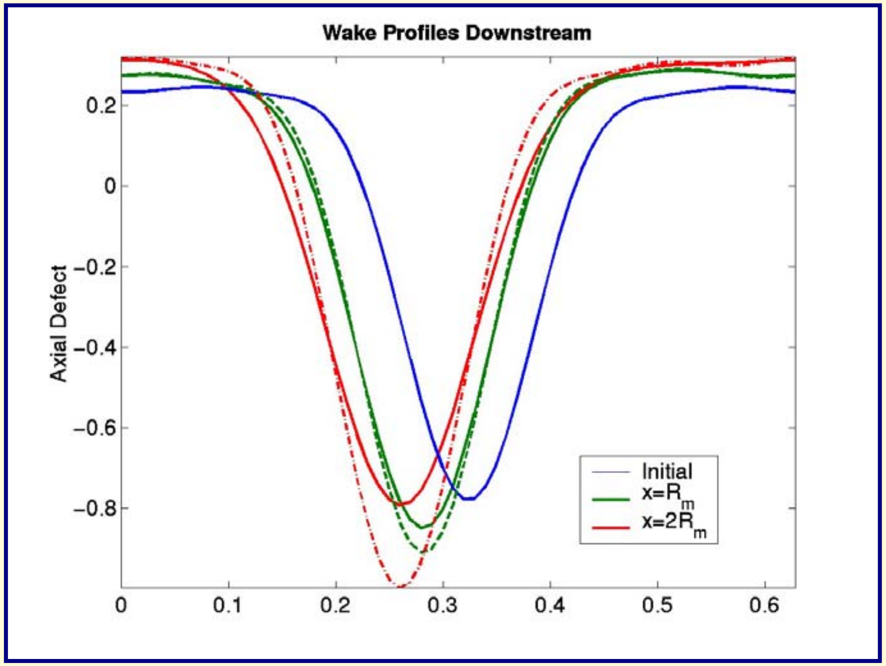
Effect of viscosity

- **Small scales are most affected by viscosity.**
  - **For large modal number *m (equivalent to wave-number)*, viscous effects are large.**
- **Rapid-distortion theory assumes viscosity as a source term modifying the evolution process.**
  - **Slip/Non-slip boundary conditions were tested.**

Effect of Reynolds number on the modes

$$\mathbf{U}(x, r) = 85\,\mathbf{e}_x + \frac{50}{r}\mathbf{e}_\theta,\ m = 10 \text{ and } m = 20,\ \omega = 5000$$





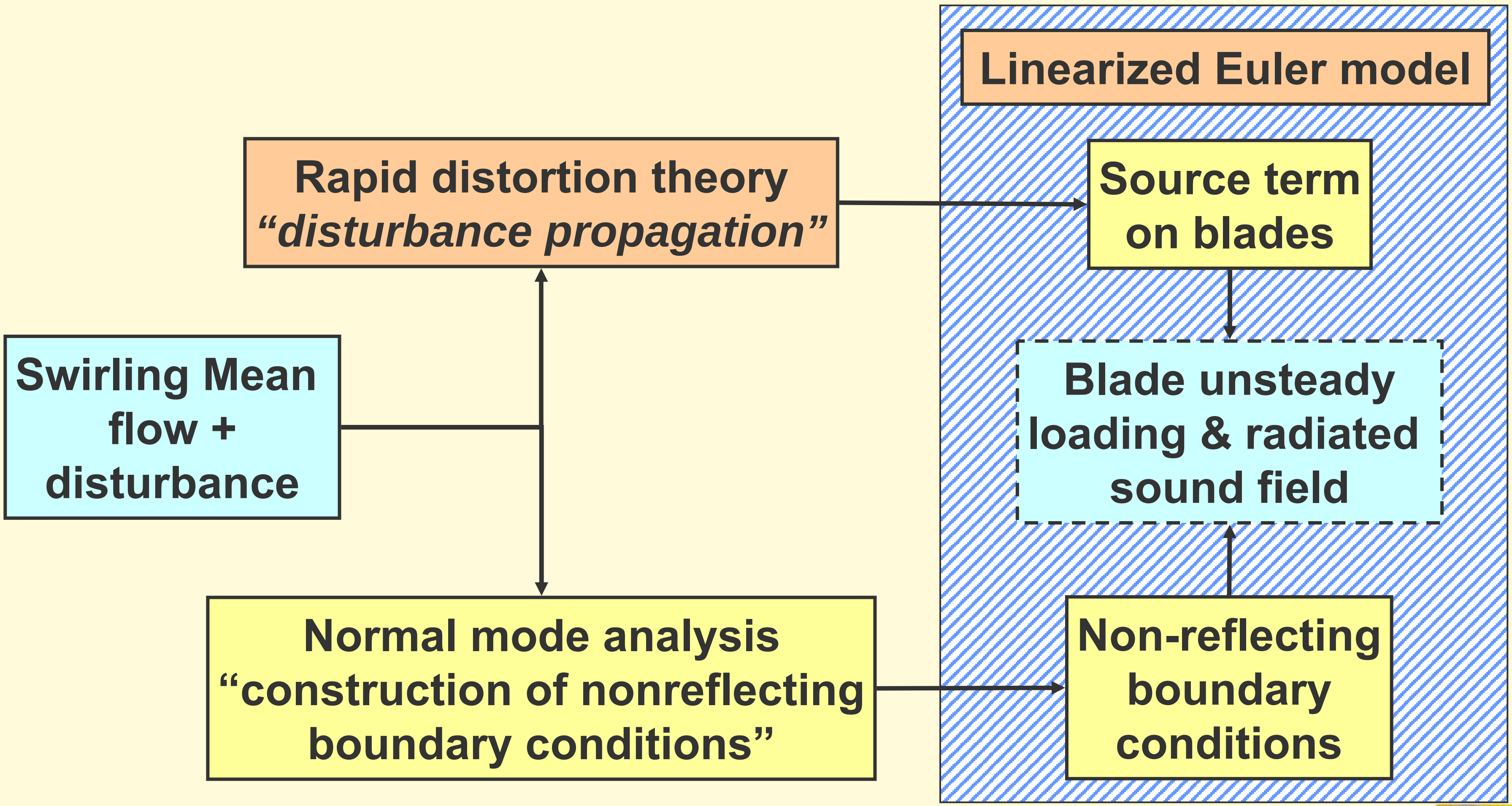
- **Re=10,000**

$$\textbf{Damping} \equiv \chi \approx O\left(\exp\left(-\frac{\beta m^2}{r^2}x\right)\right)$$

$$\beta = \frac{1}{Re_t\,\rho_o U_o}$$

Aerodynamic and Acoustic Blade Response

Aerodynamic and Acoustic Blade Response



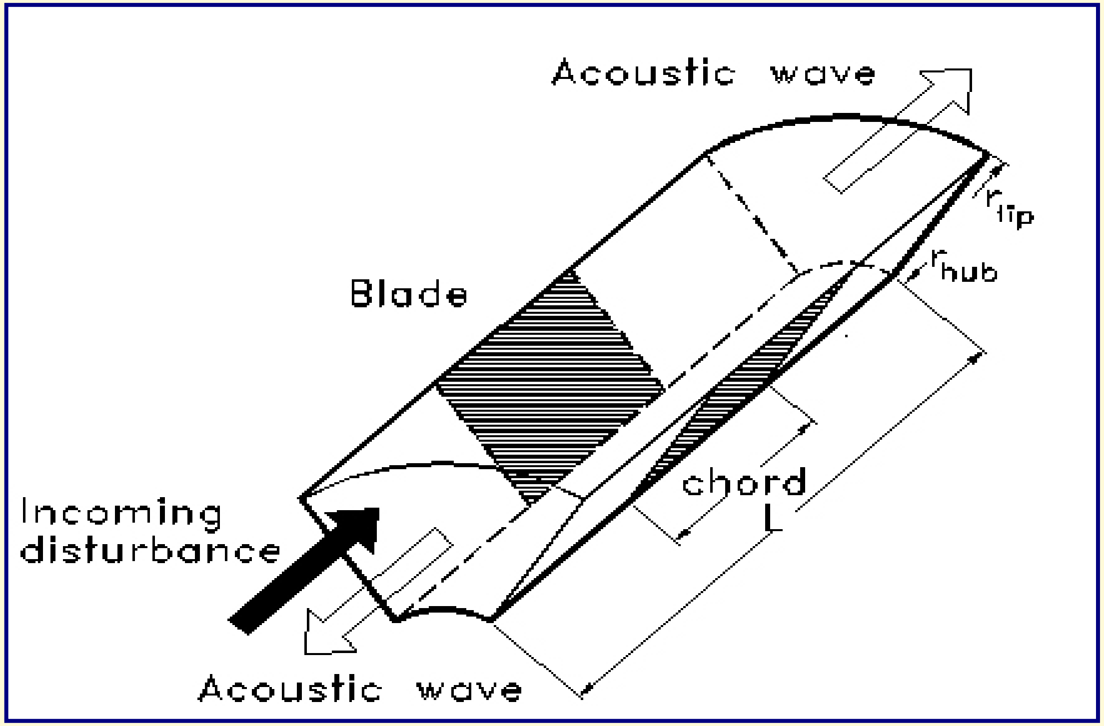
Linearized Euler Model

- **Two schemes are developed:**
  - **Primitive variable approach**
    - Pseudo Time Formulation.
    - Lax-Wendroff Scheme.
  - **Splitting velocity field approach**
    - Help understand physics.
    - Computational time requirements reduced.
    - No singularity at leading edge.
    - Implicit scheme leads to large number of equations which must be solved using an iterative method.
    - Parallelization significantly reduces computational time.

Benchmark Test Problem

$$v_\theta(r,\theta,x) = \alpha U_x e^{i(\omega x + B\theta + h(r))}$$

$$h(r) = \frac{-2\pi q}{B}\left(\frac{r - r_{hub}}{r_{tip} - r_{hub}}\right)$$



Parameters for Benchmark Test Problem

Narrow Annulus		Full Annulus		Data	
r_{tip}/r_{hub}	1.0/0.98	r_{tip}/r_{hub}	1.0/0.5	M_x (mach number)	0.5
ω	6.17 6.86 7.55 10.29	ω	5.64 6.26 6.89 9.40	α (disturbance)	0.1
				B (rotor blades)	16
				V (stator blades)	24
				C (chord)	$2\pi/V$
				L (length)	3c

Primitive Variable Approach

- Linearized Euler Equations
- Pseudo Time Formulation

$$\left([I]\left(\frac{\partial}{\partial t} - i\omega\right) + [A_x]\frac{\partial}{\partial x} + [B_\theta]\frac{1}{r}\frac{\partial}{\partial \theta} + [C_r]\frac{\partial}{\partial r} + [D] \right) Y = 0$$

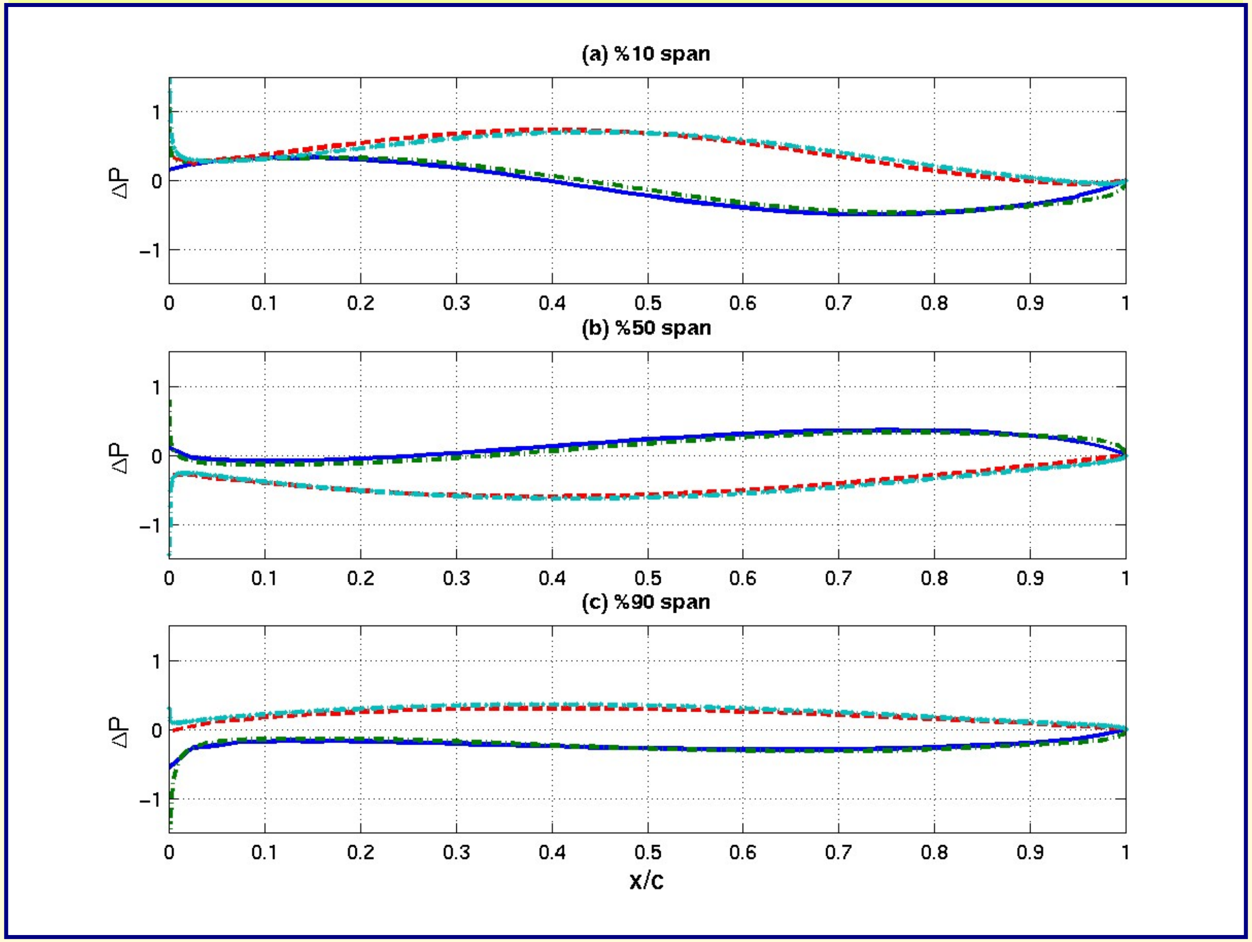
$$Y = [\rho' \quad u_x \quad u_\theta \quad u_r \quad p']^T$$

- Lax-Wendroff Scheme

Unsteady Pressure Jump Across the Blade for q=1 at Different Spanwise Locations

Primitive Variable Approach

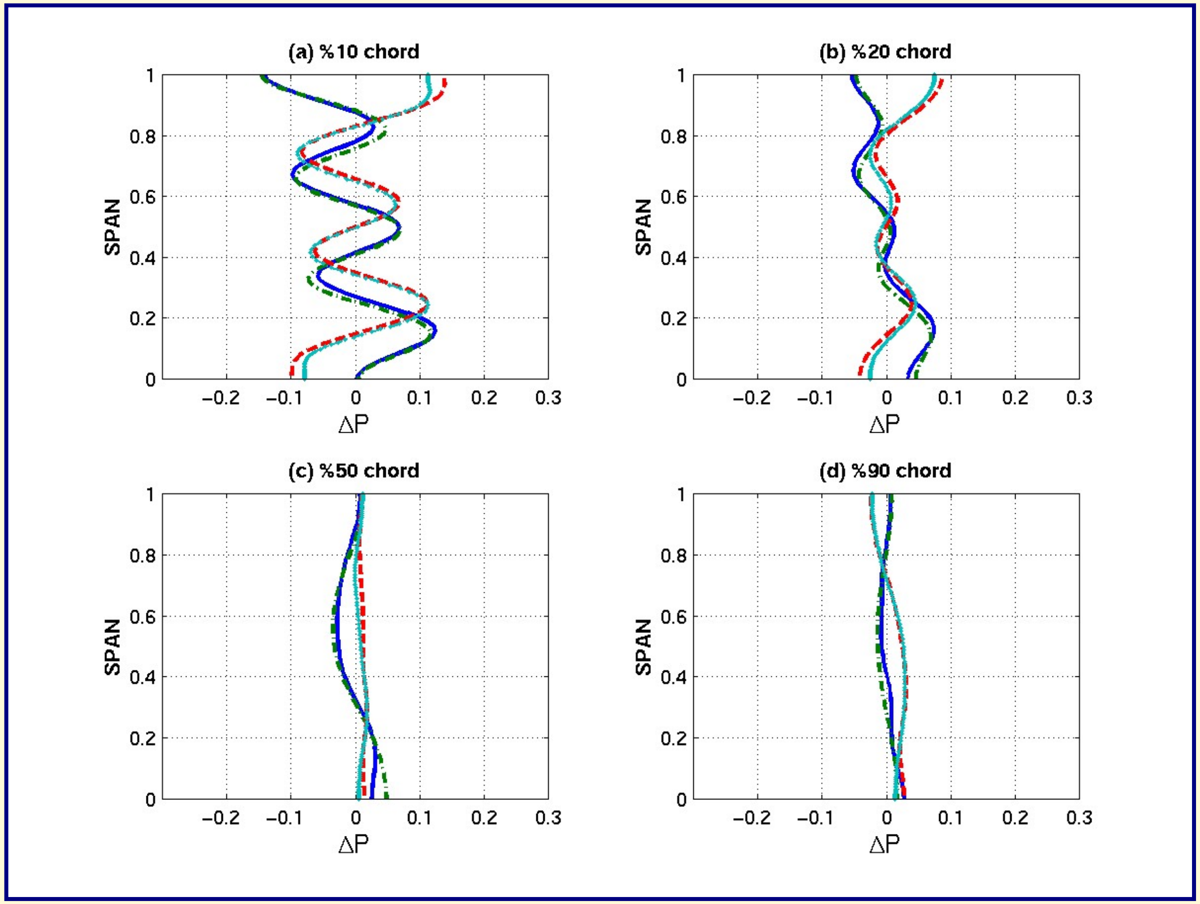
ND: real part -, imaginary part --;   Schulten: real part -.-, imaginary part …



Unsteady Pressure Jump Across the Blade for q=3 at Different Chordwise Locations

Primitive Variable Approach

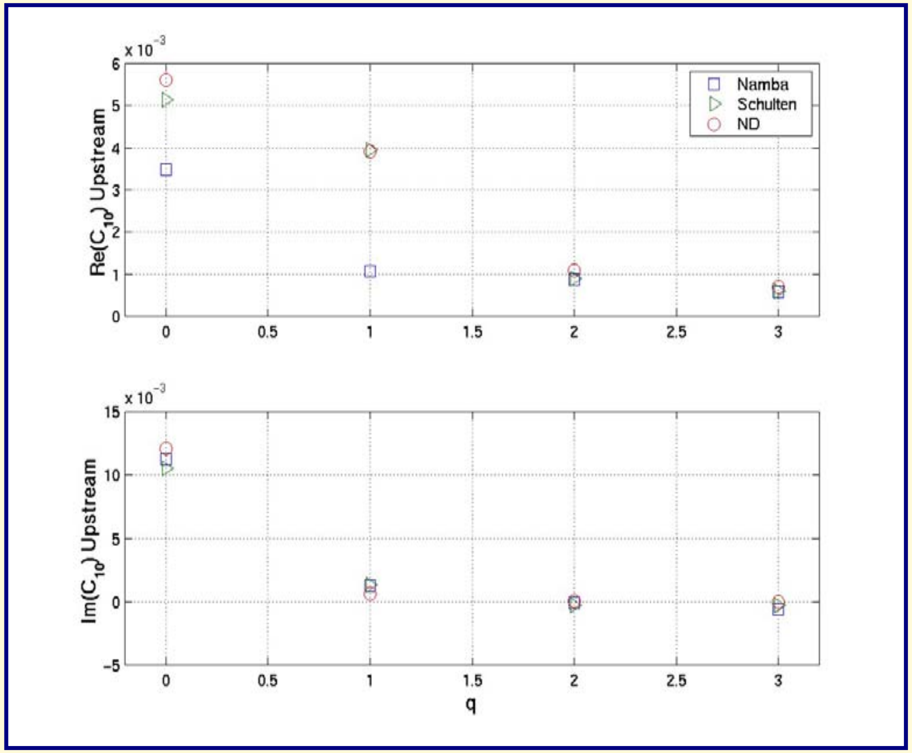
ND: real part -, imaginary part --;   Schulten: real part -.-, imaginary part …



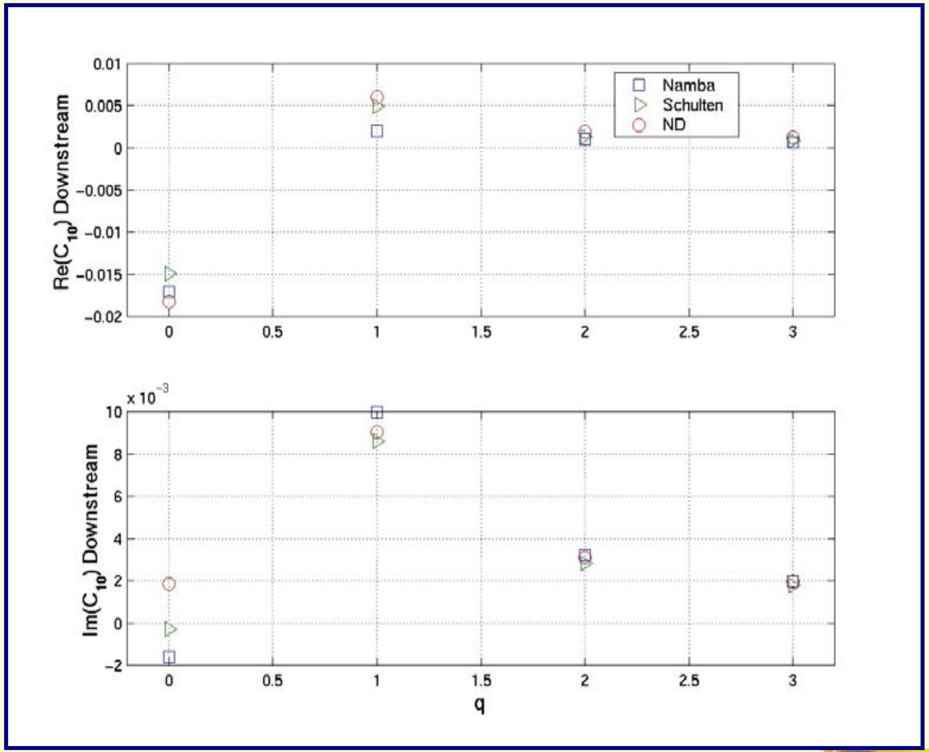
Acoustic Coefficients for Mode (1,0) at Different Gust Spanwise Wavenumbers

Primitive Variable Approach

Upstream



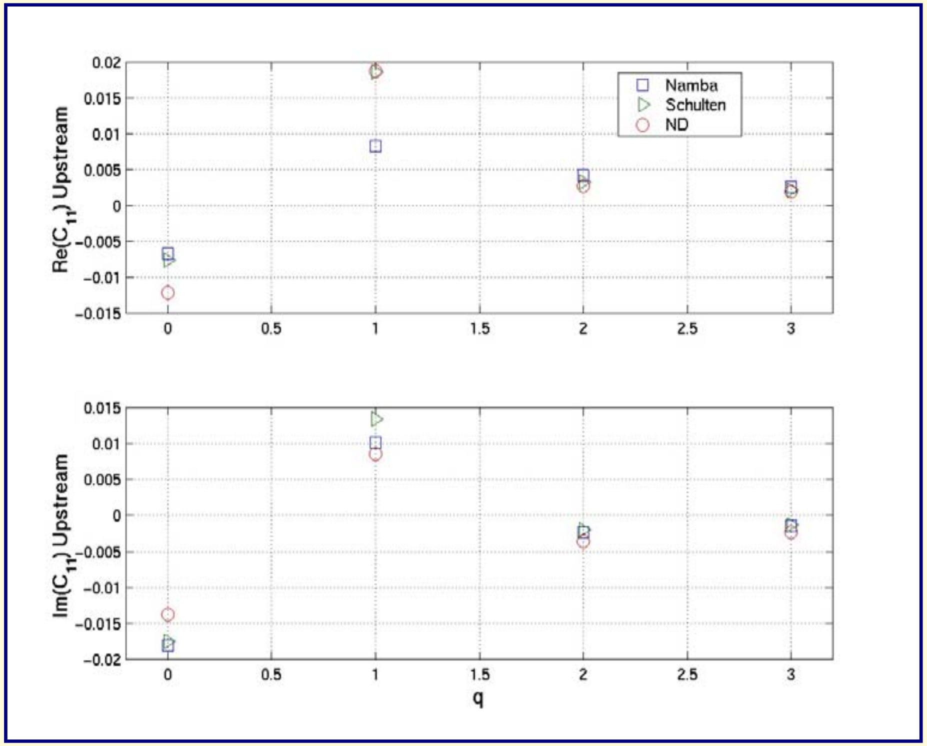
Downstream



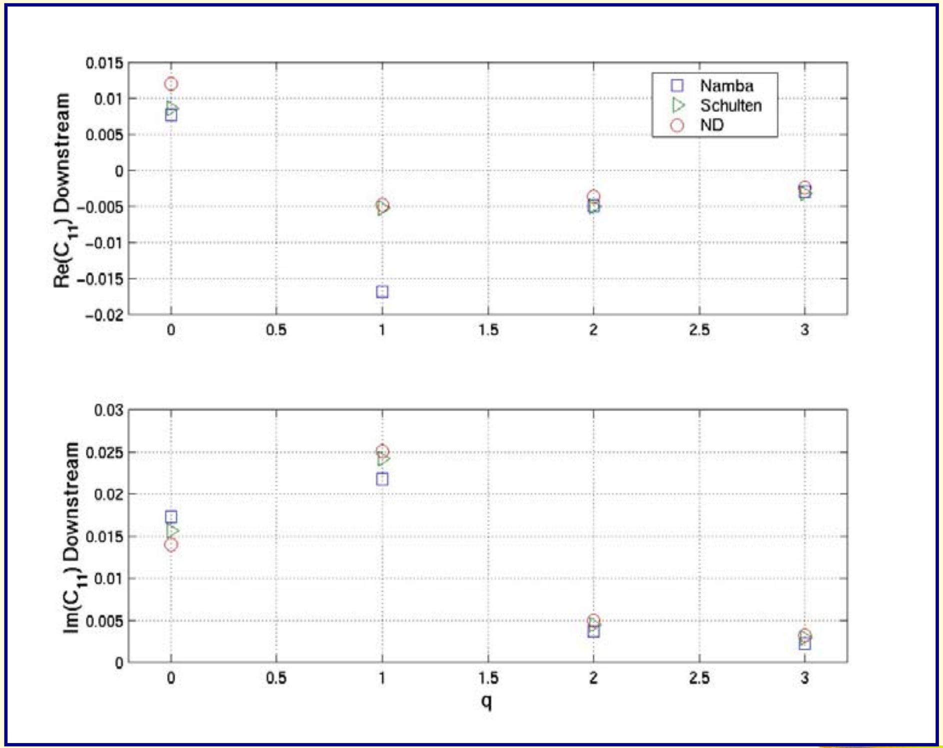
Acoustic Coefficients for Mode (1,1) at Different Gust Spanwise Wavenumbers

Primitive Variable Approach

Upstream



Downstream



Magnitude of the Downstream Acoustic Coefficients

Primitive Variable Approach

q	k	μ	Namba	Schulten	ND
0	1	0	1.7144E-02	1.4972E-02	1.8328E-02
0	1	1	1.8946E-02	1.7850E-02	1.8413E-02
1	1	0	1.0155E-02	9.9075E-03	1.0863E-02
1	1	1	2.7500E-02	2.4696E-02	2.5465E-02
2	1	0	3.3653E-03	3.0988E-03	3.6577E-03
2	1	1	6.0722E-03	6.6977E-03	6.1183E-03
3	1	0	2.0496E-03	1.9710E-03	2.3436E-03
3	1	1	3.7287E-03	4.2455E-03	3.9937E-03

Magnitude of the Upstream Acoustic Coefficients

Primitive Variable Approach

q	k	μ	Namba	Schulten	ND
0	1	0	1.1780E-02	1.1745E-02	1.3332E-02
0	1	1	1.9301E-02	1.9064E-02	1.8358E-02
1	1	0	1.6870E-03	4.1793E-03	3.9596E-03
1	1	1	1.3088E-02	2.2913E-02	2.0612E-02
2	1	0	8.9005E-04	9.4530E-04	1.0867E-03
2	1	1	4.8305E-03	3.8368E-03	4.4787E-03
3	1	0	5.8400E-04	6.5845E-04	7.1097E-04
3	1	1	3.0332E-03	2.6001E-03	2.9529E-03

Splitting Velocity Approach

$$\mathbf{u} = \mathbf{u}^R + \nabla\phi$$

$$p' = -\rho_o \frac{D_o \phi}{Dt} \quad \text{where} \quad \frac{D_o}{Dt} = \frac{\partial}{\partial t} + \mathbf{U}_o \cdot \nabla$$

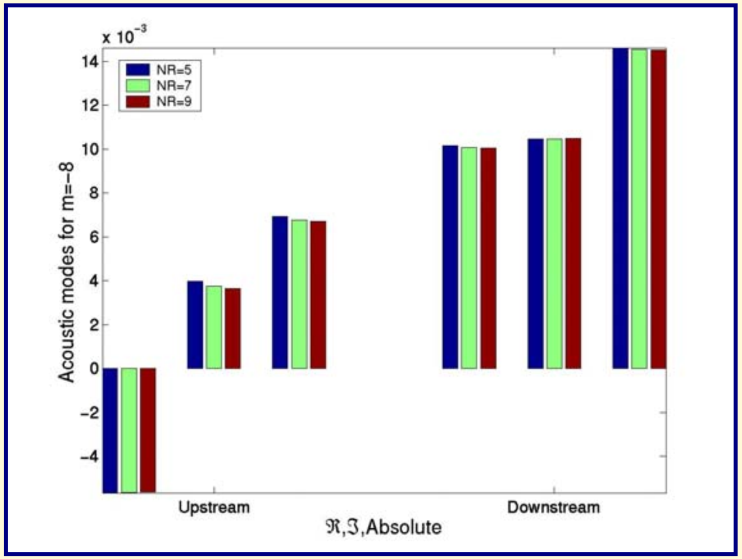
$$\frac{D_o}{Dt}\frac{1}{c_o^2}\frac{D_o\phi}{Dt} - \frac{1}{\rho_o}\nabla\cdot\left(\rho_o \nabla\phi\right) = \frac{1}{\rho_o}\nabla\cdot\left(\rho_o \vec{u}^R\right) - \frac{\partial s'/\partial t}{2c_p}$$

$$\frac{D_o \vec{u}^R}{Dt} + \left(\vec{u}^R\cdot\nabla\right)\vec{U} = -\left(\nabla\times\vec{U}\right)\times\nabla\phi - \frac{D_o\phi}{Dt}\frac{\nabla s_o}{c_p}$$

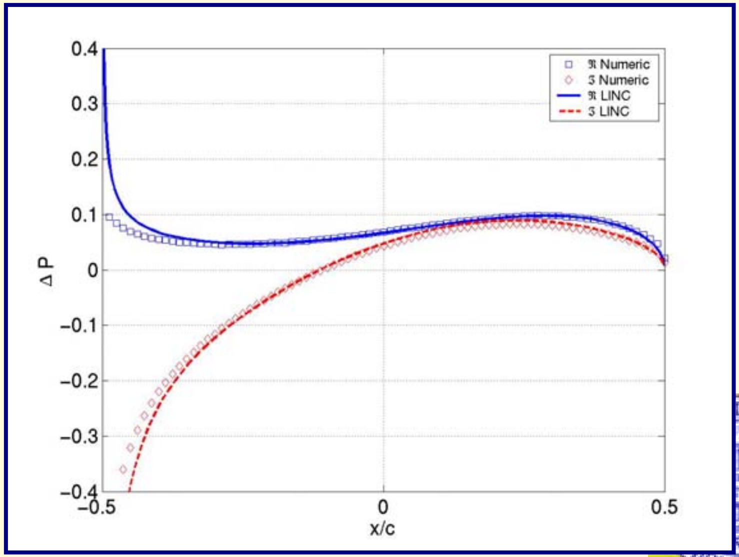
$$\frac{D_o S'}{Dt} + \left(\vec{u}'\cdot\nabla\right)s_o = 0$$

Narrow Annulus

- ω_{rm}=7.55
- Grid sensitivity study
- Pressure difference compared to LINC.

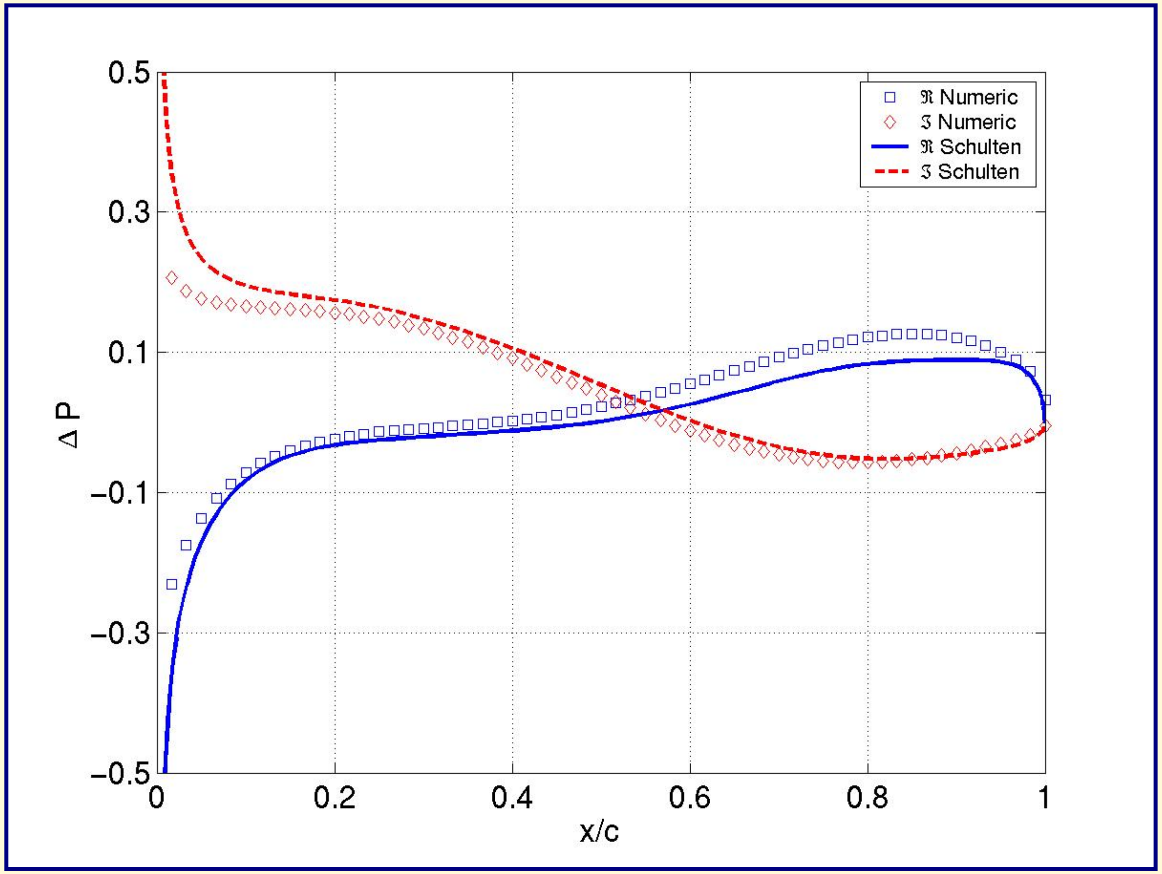


Upstream		
m=-8	Namba	$7.58x10^{-3}-1.81x10^{-3}i$
	Schulten	$7.36x10^{-3}-2.453x10^{-3}i$
	ND	$7.03x10^{-3}-3.86x10^{-3}i$
Downstream		
m=-8	Namba	$-1.12x10^{-2}+5.68x10^{-3}i$
	Schulten	$-9.95x10^{-3}+5.87x10^{-3}i$
	ND	$-9.67x10^{-3}+6.58x10^{-3}i$



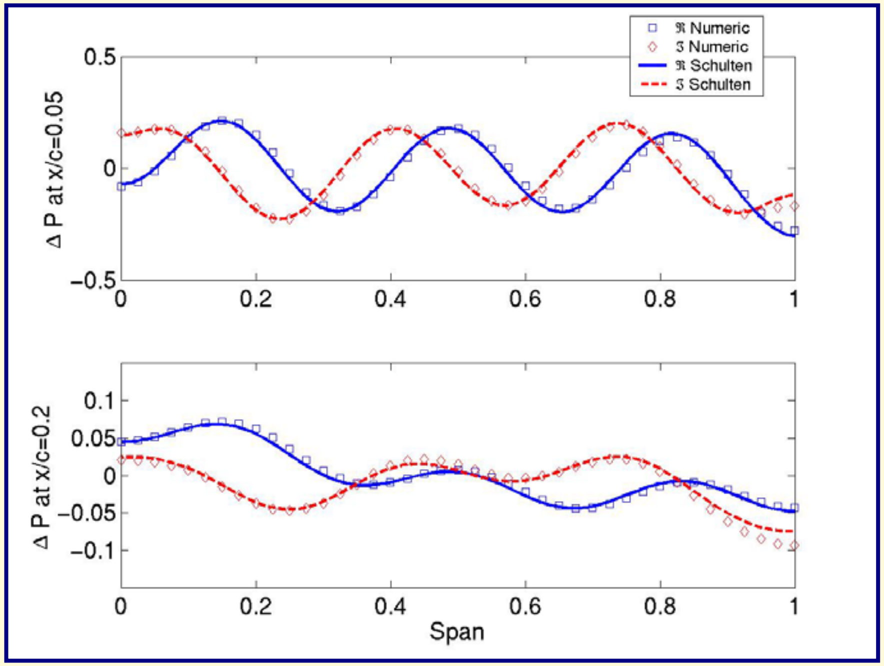
Full Annulus Case: Pressure jump for q=0, ω_{rm}=9.396. Comparison with Schulten

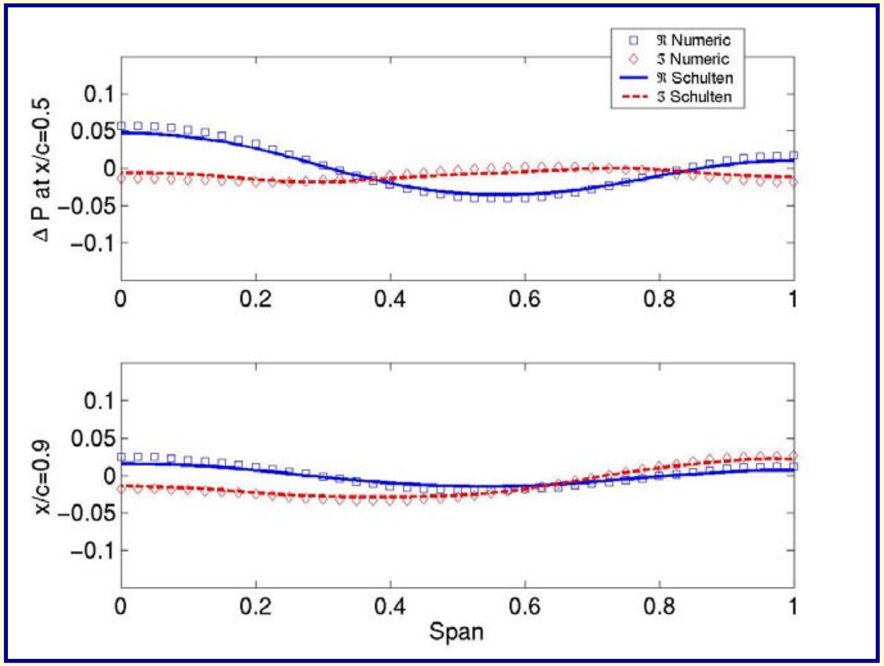
Splitting Approach



Spanwise Pressure Jump for q=3, ω_{rm}=9.396. Comparison with Schulten

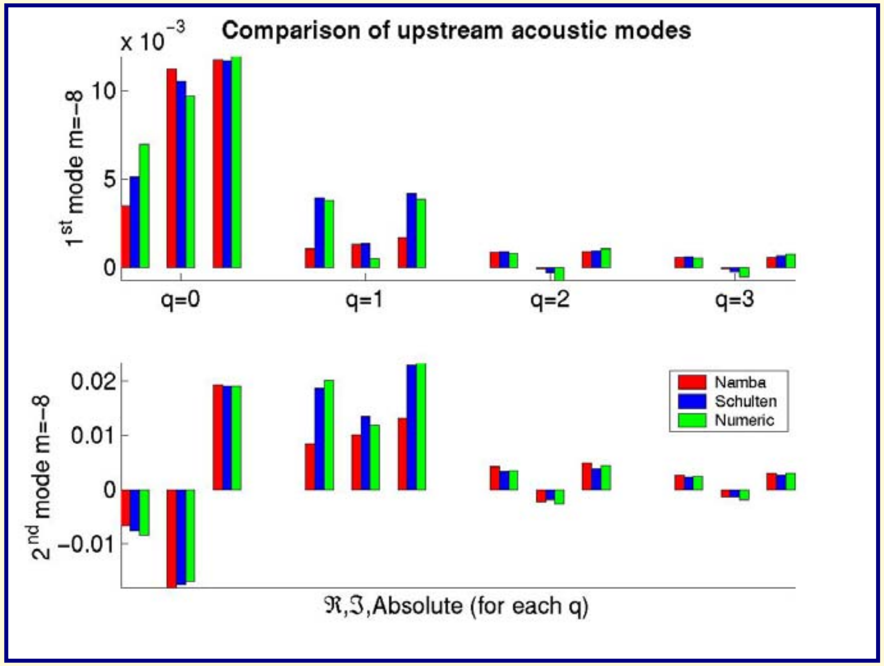
Splitting Approach

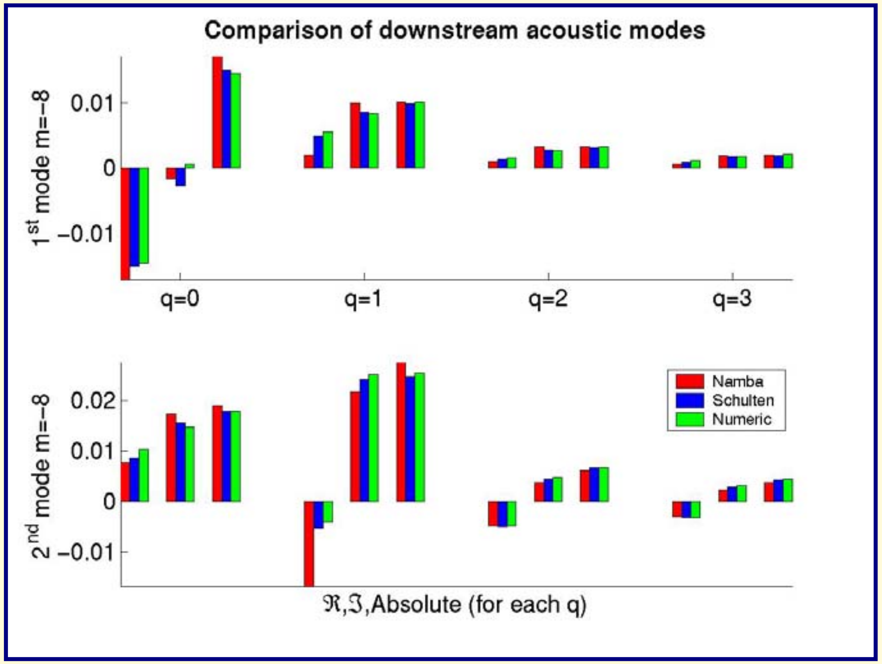




Upstream & Downstream Acoustic Coefficients.

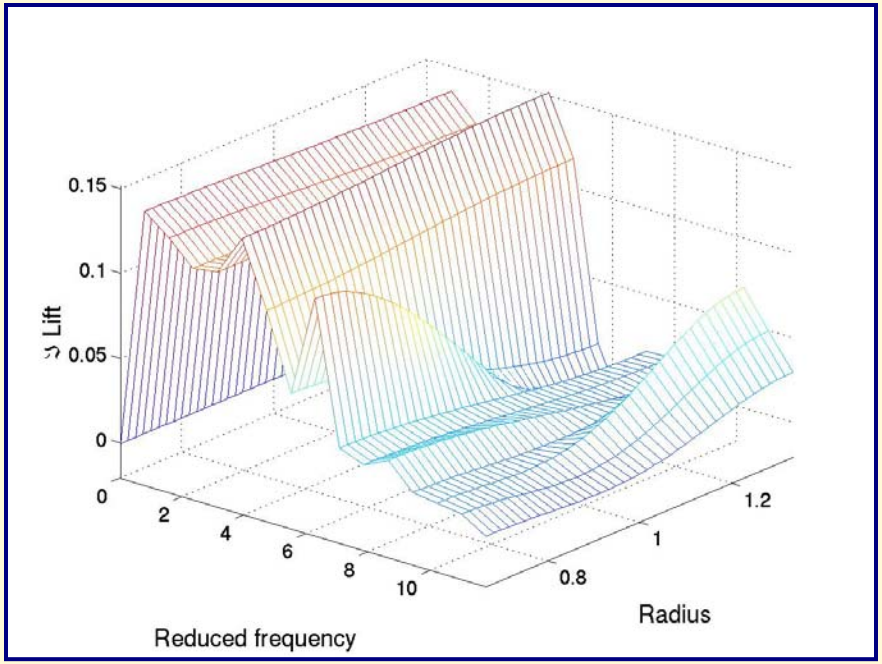
Splitting Approach

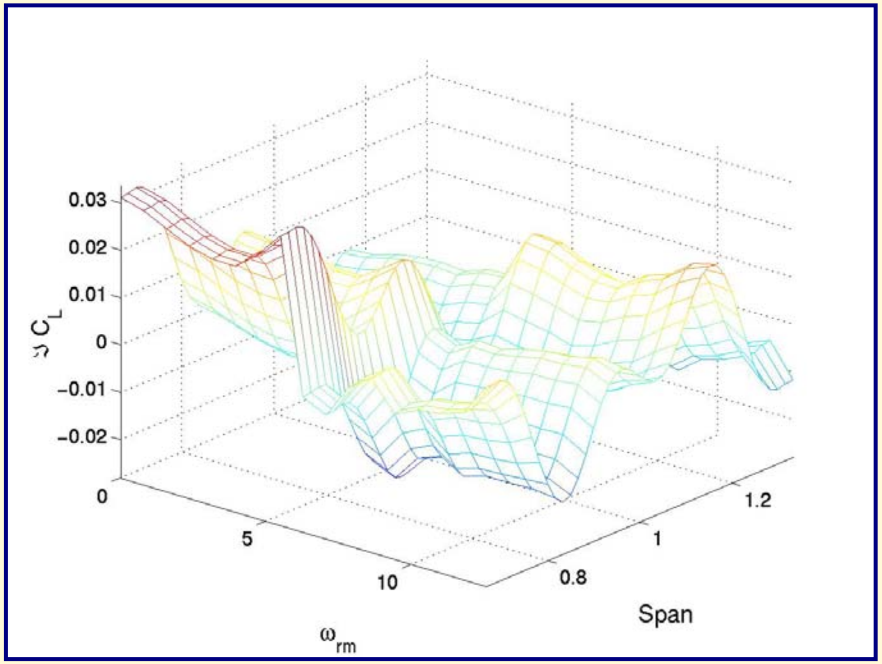




Lift Coefficient for q=0, 3 versus ω and radius

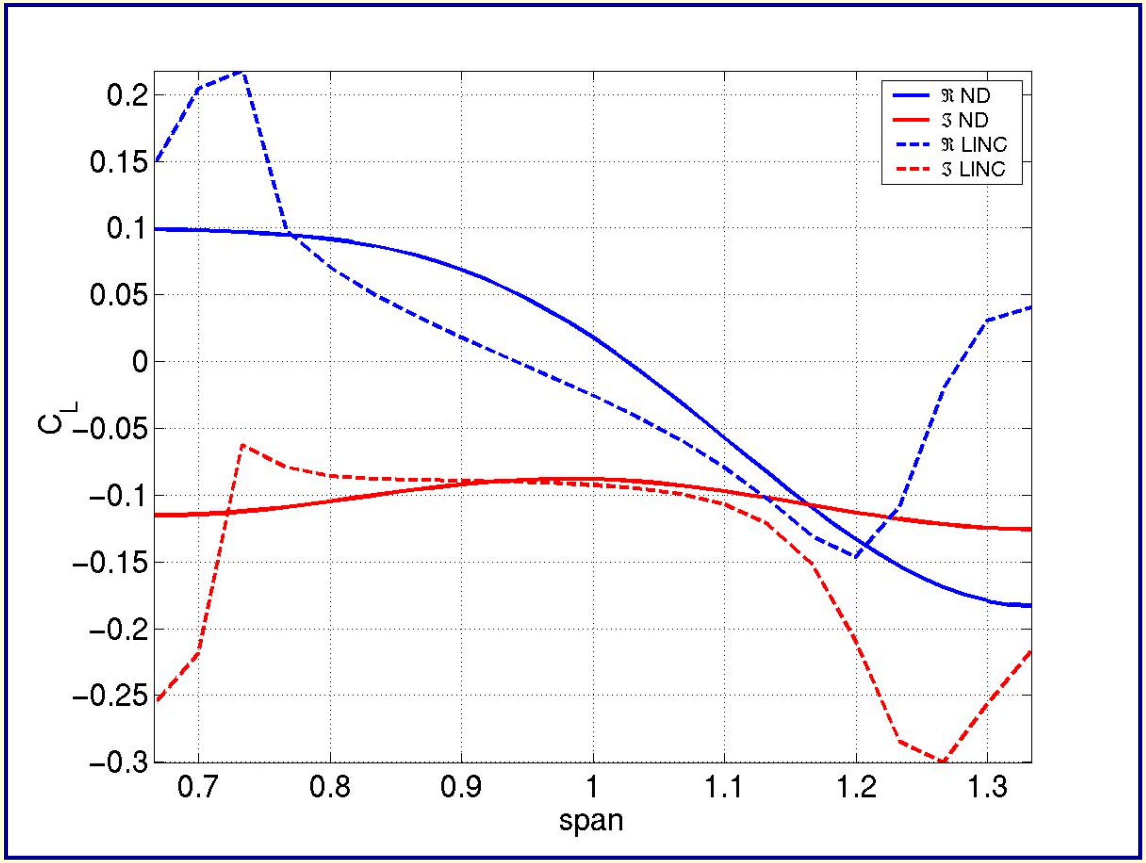
Splitting Approach



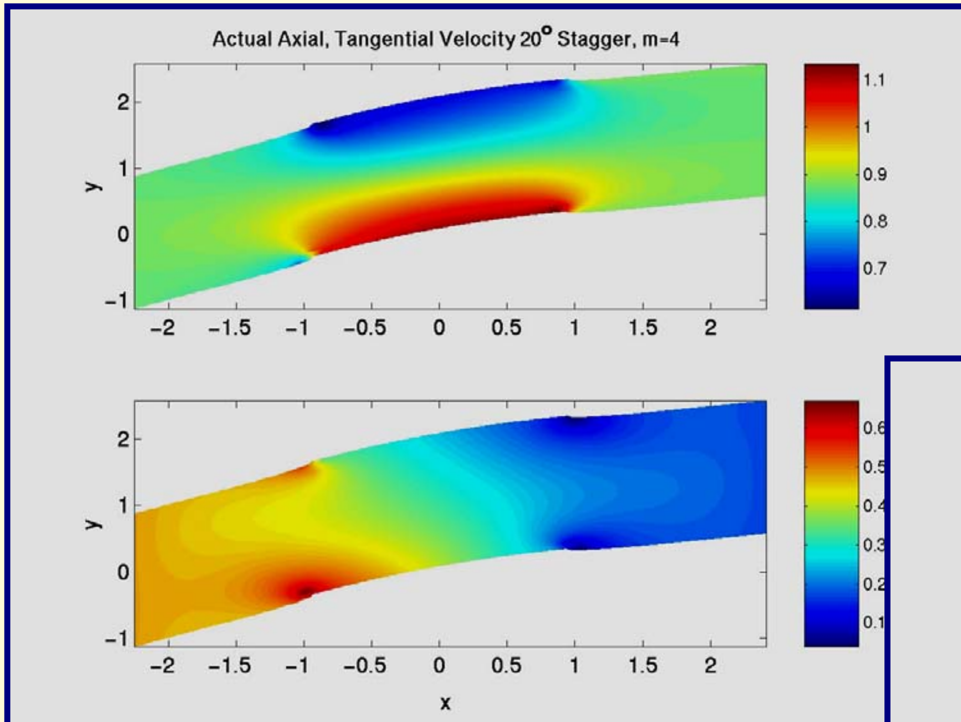


Full Annulus Lift Distribution Comparison with Strip Theory

Splitting Approach



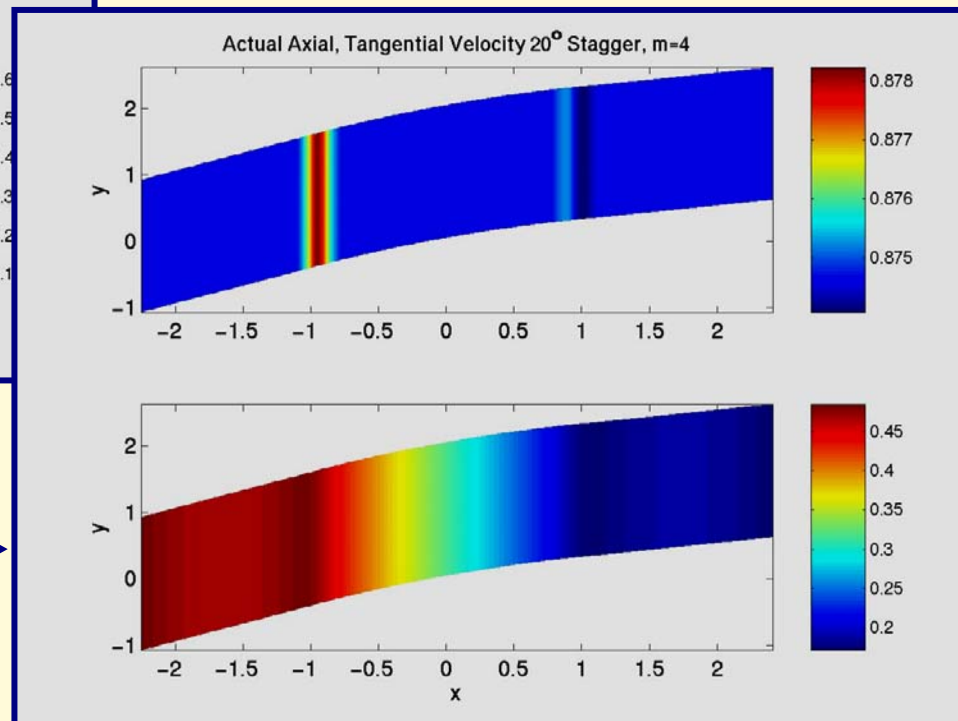
Meridian Plane Approximation for Mean Flow (2D Cascade)



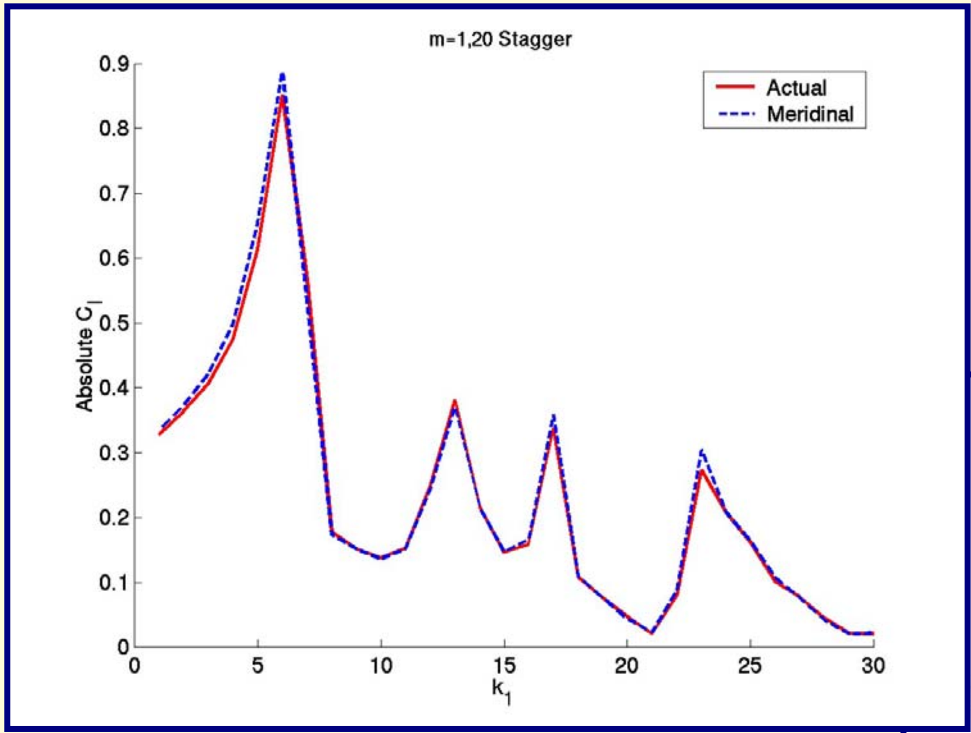
Actual Meanflow

20º stagger, M=0.3

Meridianal Meanflow

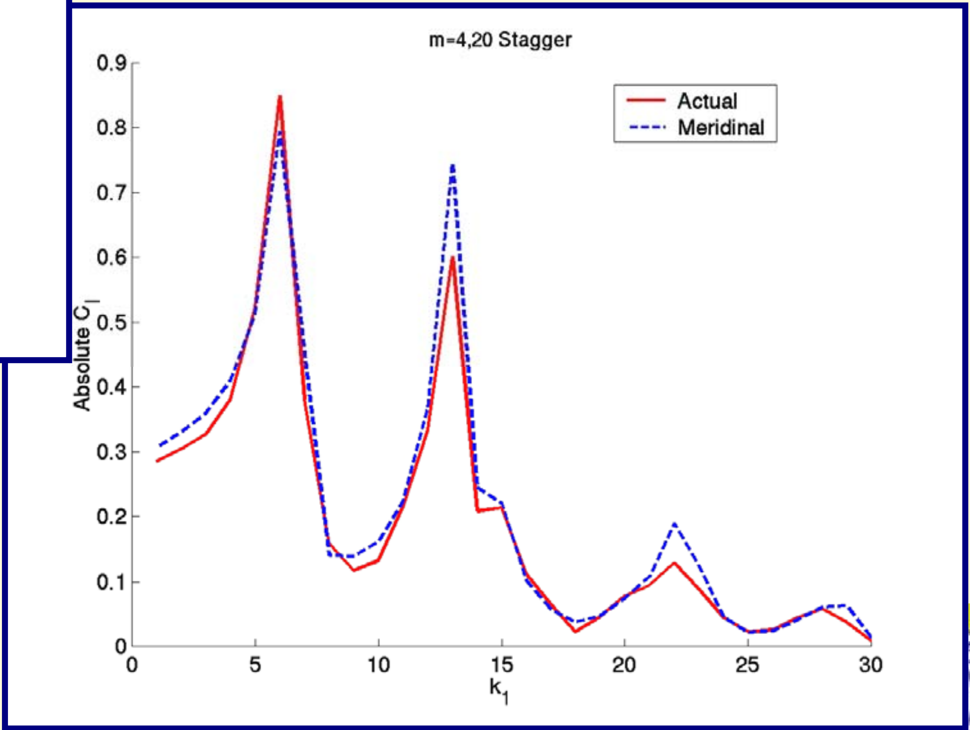


Unsteady Lift Comparison
Actual and Meridianal Meanflows



Low Loading C_l=0.20

High Loading C_l=0.92



Conclusions

- For swirling flows, two families of normal modes exist: pressure-dominated nearly-sonic, and vorticity-dominated nearly-convected modes.
- Nonreflecting boundary conditions were derived, implemented, and tested for a combination of acoustic and vorticity waves.
- An initial-Value formulation is used to calculate incident gusts.
- Two schemes (primitive variable and splitting) have been developed for the high frequency aerodynamic and acoustic blade response. Results are in good agreement with boundary element codes.
- A meridian approximation of the mean flow gives “surprising” good unsteady results for 2D cascades.

Future Work

- The numerical code will be used to study unloaded annular cascades in swirling flows.
- Method is under development for loaded annular cascades in swirling flows using a meridian approach.
- Parallelization will significantly reduce computational time making it possible to treat broadband noise.
- Express results in term of the acoustic power radiated.