Multiuser Detection Based on a Deterministic Error Specification: Theory and Low-Complexity Adaptive Solutions

Shirish Nagaraj, Sridhar Gollamudi, Samir Kapoor, Yih-Fang Huang *
Department of Electrical Engineering
University of Notre Dame, Notre Dame, IN 46556
huang.2@nd.edu

J. R. Deller, Jr.
Department of Electrical Engineering
Michigan State University, East Lansing, MI 48824
deller@egr.msu.edu

Abstract

This paper proposes a new strategy for multiuser detection in DS/CDMA systems. The detectors are designed to meet a deterministic worst-case error performance measure. Formulations with and without the use of training signals are considered and a solution for each is presented. Further, they are shown to be near-far resistant. An advantage in posing the multiuser detection problem this way is that computationally efficient adaptive algorithms can be derived for both the blind and non-blind cases, whose performance is very comparable to that of traditional RLS-like algorithms. It is also shown that sharing of detectors across users is possible, leading to significant reduction in hardware requirements.

1. Introduction

Multiuser detection (MUD) for Direct-Sequence Code Division Multiple Access (DS/CDMA) systems deals with detection of a desired user in the presence of interfering users and ambient noise. The principles of MUD are well established and understood [12]. An optimal maximum likelihood solution was found in [12], whose complexity was exponential in the number of users. Much attention has been focussed on linear multiuser detectors in which class, the decorrelating detector and the MMSE detector were derived [5, 6, 11]. The need to know the parameters of the interfering users (timing, signature sequence, received amplitudes etc) is alleviated by adaptive schemes to achieve these goals. However, since these adaptive methods require the use of training bits, blind adaptive multiuser detection is proposed in [3, 9]. A bottleneck arises in these adaptive detectors due to the trade-off between achievable performance and computational complexity. Methods to overcome this difficulty have been proposed [10].

Section 2 establishes a new performance criterion for MUD by imposing a worst-case deterministic error specification on the detector. This type of performance measure has been used in channel equalization [1, 2]. Solutions for both blind and non-blind MUD are derived when the interference parameters are known. These are shown to be equivalent in a certain sense and are also near-far resistant [12]. Section 3 discusses adaptive solutions involving projection-based algorithms. Some convergence properties of the recursive estimates are also discussed. Section 4 contains simulation results and a discussion of the performance, and Section 5 concludes the paper.

2. The Criterion for Multiuser Detection

For simplicity, we shall consider a synchronous CDMA network of K users. It is pertinent to note that a K-user asynchronous system can be conceptually viewed as an equivalent synchronous system with $(2K-1)$ users [8]. Throughout this paper, we assume that user 0 is the desired user to be demodulated. In a synchronous setup, "one-shot" detection is possible. That is, symbols are detected based on the received signal in one symbol period (T). The received

*This work was supported, in part, by the National Science Foundation under Grant MIP-9705173, in part by the Center for Applied Mathematics, University of Notre Dame, and in part, by the Tellabs Research Center, Mishawaka, IN.

signal in the interval $[iT, iT+T)$, is given by

$$r(t) = A_0 b_0(n) s_0(t-iT) + \sum_{k=1}^{K-1} A_k b_k(i) s_k(t-iT) + n(t) \quad (1)$$

in which $A_k, b_k(i), s_k(t)$ are the received amplitude, i^{th} symbol and signature sequence of the k^{th} user, respectively. We further assume that *"short codes"* are used, although this assumption is needed only in Section 3, where adaptive algorithms are discussed. This implies that the signature sequences are periodic with a period equal to the spreading gain, N:

$$s_k(t) = \sum_{l=0}^{N-1} a_k(l)\psi(t - lT_c); \quad T = NT_c; \quad t \in [0, T)$$

and $s_k(t-iT) = s_k(t) \quad \forall t, i.$ $\psi(t)$ is a rectangular chip waveform of duration T_c and $\{a_k\}$ is a binary-valued spreading code. The received signal is chip-matched filtered and sampled at the chip-rate $(1/T_c)$, which yields a discrete-time model for the signal as follows:

$$\mathbf{r}(i) = (AS)^T \mathbf{b}(i) + \mathbf{n}(i) \quad (2)$$

where $\mathbf{r}(i)$ is a $N \times 1$ vector of the sampled received values, $S^T = [\mathbf{s}_0, \ldots, \mathbf{s}_{K-1}]$ is an $N \times K$ matrix of the spreading sequence and $A = \text{diag}[1, A_1, \ldots, A_{K-1}]$ is the $K \times K$ matrix of received amplitudes (without loss of generality, we have normalized the amplitudes such that $A_0 = 1$). Also $\mathbf{b}(i) = [b_0(i), \ldots, b_{K-1}(i)]^T$ is the $K \times 1$ vector of bits of the K users ($b_0(i)$ is the desired bit to be detected) and $\mathbf{n}(i)$ is a $N \times 1$ vector of *i.i.d.* zero-mean Gaussian noise samples each with variance σ_n^2. A linear detector is an $N \times 1$ vector, $\mathbf{c}_0$, that linearly maps the received vector to the decision output:

$$z_0(i) = \mathbf{c}_0^T \mathbf{r}(i); \quad \hat{b}_0(i) = \text{sgn}[z_0(i)] \quad \forall i \quad (3)$$

where $\hat{b}_0(i)$ is the estimate of $b_0(i)$.

Common performance indices for the design of linear multiuser detectors include the zero-forcing (decorrelating) and the MMSE criteria. The decorrelating detector nulls out the contribution of the multiple access interferers (MAI) in the output, but results in noise-enhancement. The MMSE detector achieves a balance between MAI and noise suppression by partially suppressing the MAI.

2.1. Formulation with training

In this section, we propose a performance measure that ensures that the worst-case error achieved by the detector is bounded by a specified value for all received data that come from a certain *design space*. By doing so, we can be assured of good performance on a deterministic basis, *i.e.*, of achieving bounded errors for a large subset of the possible received signal vectors. We might also be able to design the detector so that it results in error-free decisions for a subset of the received vectors. In certain special cases, we show the equivalence of this detector to the MMSE detector and the decorrelating detector.

Let the error specification be that the maximum allowable error magnitude is set at a value $\gamma > 0$. That is, design a detector $\mathbf{c}_*$ such that

$$\sup_{b_0 \in \{-1,1\}, \mathbf{r} \in \mathcal{H}} |e| = \sup_{b_0 \in \{-1,1\}, \mathbf{r} \in \mathcal{H}} |b_0 - \mathbf{c}_*^T \mathbf{r}| \leq \gamma \quad (4)$$

where $\mathcal{H}$ is a design space consisting of all the received vectors formed by the additive noise values bounded in magnitude by some constant β_n:

$$\mathcal{H} \triangleq \{\mathbf{r} = (AS)^T \mathbf{b} + \mathbf{n}: \quad \|\mathbf{n}\|_\infty \leq \beta_n, \quad \forall \mathbf{b} \in \{-1, 1\}^K\}$$

where $\|\cdot\|_\infty$ is the vector infinity norm. Given $\mathcal{H}$, there exists a set of detectors that can achieve the error specification. This set is called the *feasibility set* $\mathcal{F}$ and we need to find any $\mathbf{c}_* \in \mathcal{F}$. In the following, a sufficient condition for the existence of a non-empty feasibility set is given. By upper bounding the error expression using (4), (2) and minimizing over all $\mathbf{c}$, it can be shown that

$$\text{If} \quad \|(AS\mathbf{c}_* - \hat{\mathbf{e}}_0)\|_1 + \beta_n \|\mathbf{c}_*\|_1 \leq \gamma, \quad (5)$$

then $\mathbf{c}_*$ is a detector that meets the error magnitude specification where $\mathbf{c}_* = \mathbf{R}^{-1}\mathbf{s}_0$, $\mathbf{R} = \left(S^T A^2 S + \frac{N\beta_n^2}{K} I\right)$ and $\hat{\mathbf{e}}_0 = [1, 0, \ldots, 0]^T$ is the unit vector of dimension $K \times 1$ ($\|\cdot\|_1$ is the vector 1-norm). Using (5), β_n can be chosen such that there is at least one detector which meets the error specification γ. It is interesting to note the following points:

- The structure of the detector is very similar to that of the MMSE detector. In fact, if (5) is met for $\beta_n = \sqrt{(K/N)}\sigma_n$, then $\mathbf{c}_*$ is the MMSE detector.
- In the noise-free case, we can let $\beta_n \to 0$ which results in $\mathbf{c}_*$ reducing to the decorrelating detector. Since it is known that the decorrelating detector is near-far resistant [5], it follows that $\mathbf{c}_*$ is also near-far resistant.
- By setting $\gamma \leq 1$, we impose the constraint that the detector make no errors whenever the received signal comes from the design space, $\mathcal{H}$.

2.2. Formulation without training

The MMSE counterpart for blind (anchored) detection is the Minimum Output Energy (MOE) criterion. We impose an analogous criterion in this framework that the worst-case output magnitude be bounded from above by a specified γ

under the constraint that the projection of the detector in the direction of the desired signature sequence be fixed at unity.

In this case, the problem is to design a detector $\mathbf{c}_*$ such that

$$\sup_{\mathbf{r}\in\mathcal{H}} |z| = \sup_{\mathbf{r}\in\mathcal{H}} |\mathbf{c}_*^T \mathbf{r}| \leq \gamma, \tag{6}$$

subject to the constraint $\mathbf{c}_*^T \mathbf{s}_0 = 1$. A sufficient condition for this to be possible is given by:

$$||AS\mathbf{c}_*||_1 + \beta_n ||\mathbf{c}_*||_1 \leq \gamma, \tag{7}$$

where $\mathbf{c}_* = \frac{1}{\mathbf{s}_0^T \mathbf{R}^{-1}\mathbf{s}_0} \mathbf{R}^{-1}\mathbf{s}_0$ and $\mathbf{R}$ is defined as before. This detector is a scaled version of the non-blind case and so achieves the same performance. This is similar to the equivalence of the MMSE and the MOE detectors [3].

3. Low-Complexity Adaptive Algorithms

The previous section outlined possible *"off-line"* solutions for the problem posed. These require knowledge of the interference parameters - an undesirable overload for implementation in real systems. In the following, adaptive algorithms are proposed for adaptation with and without training. These algorithms use a constrained least-squares-like cost function and are derived using ideas of projections onto convex sets. In each of the following cases, we will derive algorithms with automatic and variable gain assignment and sparse updating.

3.1. Case 1: Adaptation with Training

Consider the case in which training bits are used initially for convergence of the detector, after which the updating is performed in decision-directed mode. For the i^{th} symbol, we receive the vector $\mathbf{r}(i)$ and the desired output is the bit that was transmitted, $b_0(i)$. To develop a recursive algorithm, assume that $\mathbf{c}(i-1)$ is the detector at time $i-1$, obtained using the values $[\mathbf{r}(k), b_0(k)]_{k=1}^{i-1}$. This method is similar to that proposed by the authors for a general filtering problem [2, 7], namely, to construct a quadratic cost function at time i, given by

$$F(\mathbf{c}) = (\mathbf{c} - \mathbf{c}(i-1))^T \mathbf{W}^{-1} (\mathbf{c} - \mathbf{c}(i-1)), \tag{8}$$

where $\mathbf{W}$ is some positive-definite weighting matrix. The estimate at the present time is then obtained by minimizing this cost function subject to the constraint that the error be less than or equal to the error specification, γ. That is,

$$|b_0(i) - \mathbf{c}^T \mathbf{r}(i)| \leq \gamma. \tag{9}$$

This constraint describes a pair of N-dimensional hyperplanes, the interior of which is denoted by $\mathcal{O}(i) = \{\mathbf{c}: |b_0(i) - \mathbf{c}^T\mathbf{r}(i)| \leq \gamma\}$ - a convex set in $\mathcal{R}^N$. If $\mathbf{c}(i-1)$ belongs to $\mathcal{O}(i)$, then there is no need to update the estimate at time i. Otherwise, the new estimate lies on the bounding hyper-plane nearest to $\mathbf{c}(i-1)$, resulting in an *a posteriori* error magnitude of exactly γ [7]. The condition to be checked is whether $\mathbf{c}(i-1) \in \mathcal{O}(i)$, which is equivalent to checking whether $|\delta(i)| \leq \gamma$, where $\delta(i)$ is the *a priori* error, given by $\delta(i) = b_0(i) - \mathbf{c}(i-1)^T \mathbf{r}(i)$. The updating equations for an arbitrary $\mathbf{W} > \mathbf{0}$ are:

$$\begin{aligned} |\delta(i)| \leq \gamma \quad &\Rightarrow \quad \mathbf{c}(i) = \mathbf{c}(i-1) \\ |\delta(i)| > \gamma \quad &\Rightarrow \quad \mathbf{c}(i) = \mathbf{c}(i-1) + \frac{\lambda(i)\delta(i)\mathbf{W}\mathbf{r}(i)}{(1+\lambda(i)G(i))} \end{aligned} \tag{10}$$

where $G(i) = ||\mathbf{r}(i)||_{\mathbf{W}}^2$ and $\lambda(i) = \frac{1}{G(i)}(\frac{|\delta(i)|}{\gamma} - 1)$ is the time-varying gain term and also the Lagrangian multiplier in the solution of the above constrained problem.[1]

In general, the computational complexity of the algorithm above is of order N^2, but in real-time implementation, is computationally less intensive due to selective updating. By setting $\mathbf{W}$ equal to the inverse of the weighted correlation matrix of the received vectors, we obtain an RLS-type algorithm which is termed APEC-RLS (A Posteriori Error Constrained Recursive Least Squares). In this case, at time i, we have an additional update of the matrix $\mathbf{W}$ given by

$$\begin{aligned} \mathbf{W} &= \mathbf{P}(i-1) \\ \mathbf{P}(i) &= \mathbf{P}(i-1) - \frac{\lambda(i)\mathbf{P}(i-1)\mathbf{r}(i)\mathbf{r}^T(i)\mathbf{P}(i-1)}{(1+\lambda(i)G(i))}. \end{aligned} \tag{11}$$

On the other hand, by setting $\mathbf{W} = \mathbf{I}_N$, where $\mathbf{I}_N$ is the identity matrix of size N, we obtain an order N algorithm with selective updating which is termed APEC-LMS, by analogy to the Least Mean Squares algorithm [2]. At each time, the estimate in APEC-LMS is obtained such that the vector $\mathbf{c}(i) - \mathbf{c}(i-1)$ is orthogonal to the nearest bounding hyperplane [2]. This happens because the cost function (8) reduces to the Euclidean norm of the detector error. An important property of this algorithm is that at each time, the estimate moves closer (in the 2-norm) to every point in the feasibility set. Hence the sequence of errors is monotone non-increasing. It can also be shown that asymptotically, the value of the prediction error becomes less than or equal to γ in magnitude and also the algorithms cease updating.

For adaptation with training bits, the algorithms outlined above can be used to design detectors that guarantee the asymptotic worst-case error to be bounded from above by the specified γ. Recursive algorithms for blind adaptation are considered next.

[1] For some matrix $\mathbf{W} > \mathbf{0}$, the inner product between two vectors $\mathbf{x}$ and $\mathbf{y}$ with respect to $\mathbf{W}$ is denoted by $\langle \mathbf{x}, \mathbf{y} \rangle_{\mathbf{W}} \stackrel{\triangle}{=} \mathbf{x}^T \mathbf{W} \mathbf{y}$, and the norm induced by this inner product by $||\mathbf{x}||_{\mathbf{W}}^2 \stackrel{\triangle}{=} \mathbf{x}^T \mathbf{W} \mathbf{x}$.

3.2. Case 2: Blind Adaptation

Ideas similar to those used for deriving the above algorithms are employed in the development of blind recursive algorithms with an output specification. For blind adaptive MUD with an MMSE criterion, an LMS algorithm was proposed in [3], and a blind RLS algorithm has been recently proposed in [8] to suppress both MAI and narrowband interference. A blind MUD receiver in conjunction with beam forming via an antenna array is proposed in [4]. This receiver uses a similar criterion for adaptive blind MUD to the one used here, although a different algorithm is proposed.

In the blind case, we have two constraints to meet at each instant. First, the output magnitude must be less than γ. The other is that the component of the detector in the direction of the desired signature sequence be unity. As before, the update should be such that the cost function $F(\mathbf{c})$, defined in (8), be minimized subject to these two constraints,

$$\begin{aligned} |\mathbf{c}^T\mathbf{r}(i)| &\leq \gamma \qquad (12)\\ \mathbf{c}^T\mathbf{s}_0 &= 1 \qquad (13)\end{aligned}$$

We denote the set of all detectors that satisfy (12) by $\mathcal{O}(i)$. Assume that $\mathbf{c}(i-1)$ satisfies (13), then as before, if $|\delta(i)| = |\mathbf{c}(i-1)^T\mathbf{r}(i)| \leq \gamma$, there is no need to update the estimate. However, if such is not the case, then the update should be such that $\mathbf{c}(i) \in \mathcal{O}(i) \cap \{\mathbf{c}\colon \mathbf{s}_0^T\mathbf{c} = 1\}$. The solution for $\mathbf{c}(i)$ is obtained by the method of Lagrange multipliers and is given by:

$$\begin{aligned} |\delta(i)| \leq \gamma \quad &\Rightarrow \quad \mathbf{c}(i) = \mathbf{c}(i-1) \qquad (14)\\ |\delta(i)| > \gamma \quad &\Rightarrow \quad \mathbf{c}(i) = \mathbf{c}(i-1) - \kappa(i)\delta(i)\mathbf{W}\mathbf{r}^{\perp}(i),\end{aligned}$$

where

$$\mathbf{r}^{\perp}(i) = \mathbf{r}(i) - \frac{\langle \mathbf{r}(i), \mathbf{s}_0\rangle_{\mathbf{w}}}{||\mathbf{s}_0||^2_{\mathbf{w}}}\mathbf{s}_0 \qquad (15)$$

$$\kappa(i) = \frac{(1-\frac{\gamma}{|\delta(i)|}) \quad ||\mathbf{s}_0||^2_{\mathbf{w}}}{||\mathbf{r}(i)||^2||\mathbf{s}_0||^2_{\mathbf{w}} - \langle \mathbf{r}(i), \mathbf{s}_0\rangle^2_{\mathbf{w}}}. \qquad (16)$$

Here, $\mathbf{r}^{\perp}(i)$ is the component of $\mathbf{r}(i)$ that is orthogonal to the desired signature $\mathbf{s}_0$, *i.e.*, $\mathbf{s}_0^T\mathbf{W}\mathbf{r}^{\perp}(i) = 0$. By setting $\mathbf{W} = \mathbf{I}_N$, we obtain an order-N algorithm called APOC-LMS (A Posteriori Output Constrained Least Mean Squares). Although it is possible to obtain a corresponding RLS-like algorithm, it is not advisable to use the assignment in (11) which results in $\mathbf{P}(i)$ being a monotonically decreasing sequence of matrices. Numerical instabilities can arise because of the denominator in (16) going to zero. It is our conjecture that this difficulty can be overcome by setting $\mathbf{W}$ to be an exponentially weighted time-averaged correlation matrix of $\mathbf{r}(i)$, but this issue requires further investigation.

4. Simulation Studies and Discussion

A 10-user synchronous DS/CMDA system with a spreading gain of 20 was simulated. The interference amplitudes were all chosen to be five times the desired user's amplitude, to simulate a severe near-far situation. The spreading sequence was generated randomly for all the users and was varied from trial to trial for all the interferers, keeping the signature sequence of the desired user invariant through the trials. This approximately models the effect of arbitrary delays in an asynchronous system. The signal-to-background noise ratio was 20 dB after despreading.

The time-averaged Signal-to-Interference+Noise Ratio (SINR) was used as a measure of performance of the adaptive algorithms. The off-line detector in (5) resulted in a SINR of 18 dB which was the same as achieved by the MMSE detector. On the other hand, a matched filter resulted in an unacceptable -8 dB SINR. The plots of the performance of the proposed algorithms for adapting with training and blind are shown in Figures (1) and (2). The results are averaged over 500 trials. The time-averaged SINR at time i is given by

$$\text{SINR}(i) = \frac{\sum_{k=1}^{500}(\mathbf{c}_k(i-1)^T\mathbf{s}_0)^2}{\sum_{k=1}^{500}[\mathbf{c}_k(i-1)^T(\mathbf{r}_k(i) - b_{0,k}(i)\mathbf{s}_0)]^2},$$

where the subscript k indices the k^{th} trial.

In Figure (1), training bits were used for the first 500 symbols after which adaptation switched to a decision-directed mode. The value of γ was set to 0.4. The APEC-RLS updated in approximately 110 symbols out of the total of 1000 while the APEC-LMS updated about 370 times. It can be seen that the APEC-LMS and the standard LMS (with step size = 0.001) have not achieved steady-state in 500 symbols resulting in a degradation in performance in the decision-directed mode. On the other hand, the APEC-RLS performs comparably to the traditional RLS (with forgetting factor of 0.995) while using only 10$\%$ of the symbols for updating. In Figure (2), it can be seen that the APOC-LMS does as well as the forgetting-factor RLS of [8] while updating about 460 symbols out of 1000, and outperforms the traditional LMS algorithm. The value of γ here was 1. A higher γ would result in reduced updating, but there would be some loss in performance and vice-versa. It is at the designers control to adjust the value according to the trade-off between the resources available and the performance desired.

Previous experience in general adaptive filtering with these types of algorithms reveal that they are extremely adept at tracking time-varying systems. It is expected that the algorithms presented here also feature the ability to track time-variations introduced by users switching on or off. Also, it has been shown in [1] that the sparse updating can be used to advantage when multiple filters need to be adapted. Since such is the case here, where we have one detector for each user, an updator shared scheme is possible by which the number of updators needed is drastically reduced. This leads to significant savings in the hardware requirements in a practical system. For more details, refer to [1].

5. Conclusion

This paper has presented a new design criterion and resulting algorithms for linear multiuser detection in DS/CDMA systems. The detector ensures that the worst-case output error is bounded in magnitude by a designer-specified value. Solutions to this problem for systems involving training bits and otherwise were presented. Properties of these detectors were discussed, and they were shown to be near-far resistant. Connections to the MMSE and decorrelating detector were shown in special cases. Further, novel adaptive algorithms were presented which feature a selective-updating capability. It was also shown via simulations that these algorithms can serve as means to reduce the computational load while rendering excellent performance.

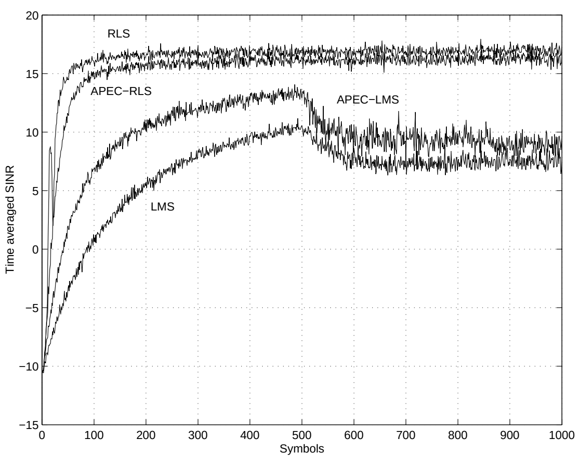

Figure 1. Performance with training

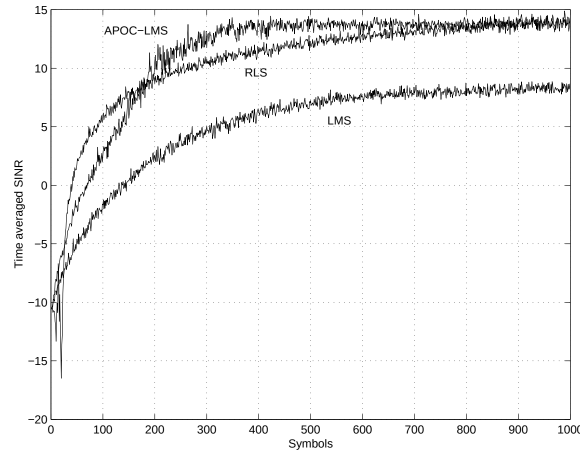

Figure 2. Performance without training

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