On the Dimensional Limitations of Linear Multiuser Detection*

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Abstract

Limitations on the interference suppression capabilities of linear multiuser detection due to dimensional constraints are studied in this paper. Generalized linear multiuser detection is defined for CDMA systems with arbitrary modulation and coding schemes and channel characteristics. Conditions are derived for the existence of linear decorrelators in this general framework and a solution for the generalized linear decorrelator is presented when it exists. This theory is applied to determine the efficacy of linear multiuser detection in important scenarios that include the presence of user asynchronism, multipath propagation, error control coding and block modulation (such as M-ary orthogonal modulation in IS-95 uplink). Ways to overcome the dimensional limitations of linear multiuser detection are to use larger symbol constellations that span a smaller number of dimensions, or to employ nonlinear techniques such as nonlinear multistage interference cancellation.

1 Introduction

The class of *linear* multiuser detection techniques for CDMA systems has attracted widespread attention in the past decade [1, 2, 3, 4], mainly due to their simplicity, amenability to adaptive implementations, and analytical tractability. Linear detectors for BPSK-modulated systems compute soft estimates of the transmitted bits as linear functions of the received signal, which are then hardlimited to make bit decisions. Since linear techniques operate by linearly separating groups of signals in the signal space, considerations on the dimensionalities of spaces occupied by desired and interfering signals are crucial in determining whether such techniques are effective in a given situation.

A generalized framework for linear detection of direct-sequence CDMA signals in the presence of multiuser interference is developed in this paper. This framework for linear detection is applicable in a variety of important scenarios such as asynchronous reception, multipath propagation, and the presence of channel coding or block modulation. Fundamental limitations of linear detection imposed by dimensional constraints in several realistic scenarios are delineated using this general framework. In this paper, linear

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detection is considered *effective* in a given scenario if a generalized linear decorrelator (GLD) exists. This analysis serves to provide a better understanding of the effectiveness of linear detectors in practical CDMA systems.

In the case of a BPSK modulated system in the presence of user asynchronism and multipath propagation, this paper shows that a linear decorrelating receiver that operates on the received signal over a *finite* interval exists if the number of users is smaller than the spreading gain. This dispels a popular notion (see, e.g., [5]) that multipath propagation increases the number of dimensions of each user's signals and consequently reduces the number of interfering signals that can be suppressed using linear multiuser detectors. However, this paper also shows that the observation interval over which the linear detector must process the received signal in order to make a decision increases rapidly with increase in multipath delay spread. If large observation intervals cannot be tolerated (due to delay, complexity or adaptivity considerations), we show that asynchronism and/or multipath propagation effectively decreases the maximum allowable users with linear reception by a factor of two. Also, whenever the number of users is smaller than half the spreading gain, it is shown that GLD's that operate over a single bit interval (*one-shot* decorrelators) exist, regardless of asynchronism and the lengths of multipath delay spreads.

This paper shows that linear decorrelating detection is possible in the presence of "linear" (or single dimensional) modulation, "non-linear" (or multi-dimensional) modulation and channel coding. The dimension of the space spanned by the code symbols is shown to determine the interference suppression capability of linear detectors. In cases where the dimension of the space spanned by BPSK modulated codewords is smaller than the number of encoded bits in each codeword (*rank deficient codebooks*), GLD's derived in this paper offer more interference suppression capability compared to the conventional linear decorrelators that do not utilize the knowledge of the code. It is shown in this paper that *with block orthogonal coding (such as M-ary orthogonal modulation used on IS-95 uplink), the number of interferers that can be suppressed by a linear detector decreases exponentially with increase in block length.*

Linear multiuser detection schemes have been suggested in literature for use with nonlinear modulation schemes (e.g. the decision projection algorithm in [6]), which are also subject to the limitations derived in this paper. In particular, the maximum number of interferers that can be suppressed using such algorithms in an M-ary orthogonal modulated system also decreases exponentially with block length. Also, the so-called "soft-decision" interference cancelling receivers, which estimate the transmitted signal of each user as a linear function of the matched filter output for cancellation [7, 8], are also linear receivers and hence are subject to the constraints presented here.

Results from this paper indicate that the number of interferers that a linear detector can suppress can be increased beyond the spreading gain if a modulation scheme is used in which the symbol waveforms span a number of dimensions smaller than the number of bits per symbol. Examples of such schemes are PAM and M-PSK. Another way to overcome the dimensional constraints of linear detection is to employ nonlinear receivers. A number of attractive nonlinear multiuser receivers such as nonlinear successive or parallel interference cancelers and decision-feedback multiuser detectors are known. Nonlinear techniques are attractive also because most of them do not require, unlike adaptive linear multiuser detectors, the spreading codes to repeat every symbol (*short codes*).

The organization of this paper is as follows. Section 2 provides a general framework for linear multiuser detection and defines a class of generalized linear decorrelating detectors. Also, in Section 2, conditions for the existence of GLD's are derived, and solutions are provided when they do exist. These concepts are used in Section 3 to derive GLD's and study the limitations of linear detection in the presence of asynchronism and multipath propagation. Section 4 analyzes how channel coding and block modulation constrain the effectiveness of linear multiuser detection, and presents GLD's (when they exist) for coded systems.

2 Generalized Linear Multiuser Detection

2.1 Preliminaries

Consider K users simultaneously transmitting signals to a central receiver in a direct-sequence CDMA system. The received signal can be represented in complex baseband notation as

$$r(t) = s_1(t) + s_2(t) + \cdots + s_K(t) + n(t), \tag{1}$$

where $s_k(t)$ is the signal received from the kth user ($k = 1, 2, \cdots, K$), and $n(t)$ is additive noise. The kth user's received signal is obtained from the following multipath model:

$$s_k(t) = A_{k1}x_k(t - \tau_{k1}) + A_{k2}x_k(t - \tau_{k2}) + \cdots + A_{kJ}x_k(t - \tau_{kJ}), \tag{2}$$

where A_{kj} and τ_{kj} are the complex-valued channel gain and delay, respectively, on the jth path. Assume, without loss of generality, that $\tau_{11} = 0$. The kth user's transmitted signal, $x_k(t)$, is a product of kth user's modulated symbol waveforms and the spreading code of user k. Let T_b denote the bit-interval in the information bit-stream of any user. The modulation and coding schemes determine the dependence of modulated symbols on the information bits of the kth user. The spreading code consists of a sequence of *chip pulses*, each spanning T_c seconds. Spreading gain N of the CDMA system is defined as the number of chip pulses in one bit interval: $N = T_b/T_c$.

Without loss of generality, our attention will be limited to detecting user 1's signals. Let $[0, T]$, where T is an arbitrary integer multiple of T_c, be a finite observation interval such that a linear multiuser detector acts on $\{r(t) : t \in [0, T]\}$ to detect the desired user's signal in that interval. Note that this interval can span several bits of the desired user: we are not restricted to *one-shot* multiuser detectors. We shall be concerned in this paper only with linear detectors that span finite intervals ($T < \infty$). It is shown in Section 3.3 that a GLD for $T < \infty$ exists if *any* GLD exists.

The received signal in the observation interval $[0, T]$ is chip-matched filtered (synchronized to the delay of user 1) to obtain a vector of chip-rate samples, $\mathbf{r}$, prior to further processing[1]. The length of $\mathbf{r}$ is equal to the number of chips in the observation interval, $N_c = T/T_c$. Let $\mathcal{M}(\cdot)$ denote the chip-matched filtering operation that linearly maps a received signal in $[0, T]$ to the corresponding N_c-tuple of chip-matched filter samples. The following concepts are required to define generalized linear multiuser detection.

$\mathcal{S}_D$: Desired signal space, spanned by all possible desired user's signal vectors at the output of chip-matched filter (in the absence of interference). If $\mathcal{D}$ is the set of all possible user 1's signals over $t \in [0, T]$, then $\mathcal{S}_D = \text{Span}\{\mathcal{M}(s_t) : s_t \in \mathcal{D}\}$. It is determined by the symbol constellation, the choice of signature codes for the desired user, and channel characteristics. Let N_D denote its dimension, and let the columns of $B_D \in \mathbb{C}^{N_c \times N_D}$ constitute a basis[2]. *Example:* In an uncoded, single

[1] For simplicity, it is assumed that the path delays of the desired user (τ_{1j}, $j = 1, 2, \cdots, J$) are integer multiples of the chip interval T_c, so that chip-matched filtering does not remove multipath information.

[2] $\mathbb{C}$ denotes the complex field.

path BPSK system with $T = T_b$, we have $N_D = 1$, and $B_D = \mathbf{c}_1$, where $\mathbf{c}_1$ is the spreading code vector of user 1.

$\mathcal{S}_I$: Interference space, spanned by the set of chip-matched filter output vectors corresponding to all possible interfering signals. In other words, if $\mathcal{I}$ is the set of all possible interfering signals over $t \in [0, T]$, then $\mathcal{S}_I = \text{Span}\{\mathcal{M}(i_t) : i_t \in \mathcal{I}\}$. This admits the possibility of chip-asynchronous interferers, since only the image of the interfering signals at the output of the desired user's chip-matched filter is of concern. Let N_I be the dimension of $\mathcal{S}_I$, and let $B_I \in \mathbb{C}^{N_c \times N_I}$ comprise a basis for it. *Example:* In a synchronous, single path CDMA system with uncoded BPSK modulation and $T = T_b$, we have $N_I = K - 1$, and $B_I = [\mathbf{c}_2 \ \mathbf{c}_3 \ \cdots \mathbf{c}_K]$, if all spreading codes are assumed to be linearly independent.

In terms of the above definitions, the received signal vector can be written as

$$\mathbf{r} = B_D\mathbf{u} + B_I\mathbf{w} + \mathbf{n}, \tag{3}$$

where $\mathbf{u} \in \mathbb{C}^{N_D}$ uniquely determines the transmitted signal of user 1, $B_I\mathbf{w}$ is the total interference signal at the output of the chip-matched filter, and $\mathbf{n} = \mathcal{M}(n(t); t \in [0, T])$. A *generalized linear multiuser detector* (GLMD) is defined by a matrix $V \in \mathbb{C}^{N_c \times N_D}$ that describes a linear mapping of $\mathbf{r}$ to an estimate of $\mathbf{u}$,

$$\hat{\mathbf{u}} = V^T\mathbf{r}, \tag{4}$$

followed by a minimum-distance detector that maps $\hat{\mathbf{u}}$ to a decision on the desired user's transmitted signal (without any knowledge of the interferers' codes). The above definition of a GLMD is extremely general, and coincides with the conventional definition of a linear multiuser detector in the case of BPSK modulated, single-path channel CDMA systems with no coding and $T = T_b$. The following sections elaborate on how this framework permits linear detection of signals in several other important scenarios.

In most prior works on linear multiuser detection, coded BPSK systems have been treated as uncoded systems by assuming that the linear detector has no knowledge of the coding scheme. In the following sections, an uncoded system refers to a system with no channel code, or a coded system in which the linear receiver estimates coded bits, as opposed to information bits using knowledge of the code. Separating linear multiuser detection and decoding operations at the receiver leads to the coding-spreading tradeoff, which is discussed further in Section 4.

2.2 The generalized linear decorrelator

Among linear multiuser detectors, of particular concern in the absence of noise is the linear decorrelating detector. A linear decorrelating detector is usually defined (see, e.g., [9, 10]) for uncoded BPSK systems with single path channels as a set of linear detectors for all K users, whose soft estimates equal the transmitted bits in the absence of noise.

In the framework of GLMD, a *generalized linear decorrelator* (GLD) for user 1 can be defined as a GLMD (cf. (4)) such that $\hat{\mathbf{u}} = \mathbf{u}$, for all possible received signals with $\mathbf{n} = 0$. In other words, the columns of V belong to the null space of $\mathcal{S}_I$, and are specified as follows[3]:

$$V^T B_D = I_{N_D \times N_D}, \tag{5}$$
$$V^T B_I = 0_{N_D \times N_I}. \tag{6}$$

[3] $I_{N \times N}$ denotes the $N \times N$ identity matrix, and $0_{N \times M}$ denotes the $N \times M$ zero matrix.

In the case of uncoded, single path BPSK, $B_D = \mathbf{c}_1$ and $\mathbf{u} = \pm 1$; therefore, the GLD specification reduces to $\mathbf{v}^T\mathbf{c}_1 = 1$ and $\mathbf{v}^T B_I = 0_{1\times N_I}$. This coincides with the conventional definition of a linear decorrelator.

The following result can be inferred from the definition of the GLD. Its proof is omitted due to lack of space.

Proposition 2.1: A necessary condition for the existence of a GLD as defined in (5) and (6) is that

$$N_D + N_I \leq N_c, \tag{7}$$

and a sufficient condition for its existence is that $B = [B_D \; B_I]$ has full column rank.

The above sufficient condition is similar to the linear independence assumption frequently used in literature on conventional linear multiuser detection. For synchronous, uncoded, single path BPSK systems with $T = T_b$, we have $N_D = 1$, $N_I = K - 1$, and $N_c = N$. Therefore, the above necessary condition reduces to the familiar dimensional limitation of the linear decorrelator: $K \leq N$.

In the remainder of the paper, we shall assume that the transmitted signals from different users are linearly independent if (7) is satisfied, so that the sufficient condition in Proposition 2.1 is satisfied whenever (7) is. This is similar to making the linear independence assumption for a conventional decorrelator. If B has full column rank, the solution set for each column of V from (5) and (6) is a subspace with $N_c - (N_D + N_I)$ dimensions. Any choice of vectors from these subspaces forms a valid GLD, but the solution in which each column of V has minimum 2-norm is given by

$$V = B(B^T B)^{-1} \begin{bmatrix} I_{N_D\times N_D} \\ 0_{N_D\times N_I} \end{bmatrix}. \tag{8}$$

Minimizing the 2-norm of the columns of V minimizes the response of the GLD to additive noise.

3 Effect of Asynchronism and Multipath Propagation

This section investigates the existence of a GLD in the presence of asynchronous reception and multipath channels. Since the GLD is the most general form of linear decorrelating receiver, the results obtained in this section characterize the fundamental dimensional limitations of linear reception in such situations.

3.1 Asynchronous users, single path channels

Consider asynchronous reception of the K users' uncoded BPSK signals through single path channels. Let the observation interval $[0, T]$ span exactly M bits of user 1, as shown in Figure 1. It follows that the number of dimensions of the desired signal space is $N_D = M$, and the number of rows of B is $N_c = MN$. A convenient basis for $\mathcal{S}_D$ is the following $MN \times M$ block-diagonal matrix:

$$B_D = \mathrm{diag}\{\mathbf{c}_1(1), \mathbf{c}_1(2), \cdots, \mathbf{c}_1(M)\} \tag{9}$$

where $N \times 1$ vector $\mathbf{c}_1(i)$ is user 1's spreading code during the ith bit interval. It follows that the desired user's signal at the output of the chip-matched filter is given by $B_D\mathbf{u}$,

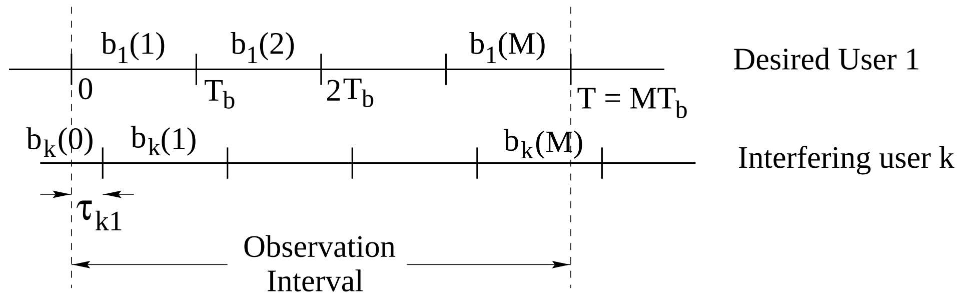

Figure 1: Asynchronous, single path reception.

where $\mathbf{u} = [b_1(1)\ b_1(2)\ \cdots\ b_1(M)]^T$. Therefore, a GLD that is obtained by using this choice for B_D in (8), if one exists, directly yields the decoded bits as its outputs.

Due to asynchronism, $M+1$ independent bits of each interfering user ($b_k(0)$ through $b_k(M)$ in Figure 1) affect the received signal, and therefore the number of dimensions in $\mathcal{S}_I$ due to each interferer is $M+1$. This implies that $N_I = (K-1)(M+1)$. Consider the matched filter output vector $\mathbf{x}_{ki}$ when user k transmits the ith bit and zero everywhere else, and all other users transmit zero everywhere. A natural basis for the interference subspace is the set $\{\mathbf{x}_{ki} : k = 2, 3, \cdots, K;\ i = 0, 1, \cdots, M\}$.

Substituting the above identities in (7), the following condition for the existence of a GLD is obtained:

$$K \leq \frac{MN+1}{M+1}, \tag{10}$$

or, equivalently,

$$K < N \quad \text{and} \quad M \geq \frac{K-1}{N-K}. \tag{11}$$

If (11) is satisfied, then the condition in Proposition 2.1 is satisfied, and a *finite length* GLD can be obtained using (8) for the asynchronous system. (11) shows that as the number of users increases from 1 to $N-1$, the observation interval must be increased from 1 to $N-2$. Also, $M=1$ is sufficient—that is, a one-shot GLD exists—if $K \leq N/2$.

3.2 Synchronous users, multipath channels

Consider synchronous uncoded BPSK transmission through multipath channels. Let $\tau_{k1} = 0$, $k = 1, 2, \cdots, K$, and let the maximum multipath channel spread be L bit periods. Let the observation interval span exactly M complete received bits, such that $N_c = (M+L-1)N$. It can be shown that the dimension of the desired signal space is $N_D = M+2(L-1)$, and the dimension of interference space is $N_I = (K-1)(M+2(L-1))$. Using these in (7), we obtain the following condition for the existence of a GLD:

$$K \leq N\left[1 - \frac{L-1}{M+2(L-1)}\right], \tag{12}$$

or, equivalently,

$$K < N \quad \text{and} \quad M \geq \frac{(L-1)(2K-N)}{N-K}. \tag{13}$$

The above condition (13) shows that for any $K < N$ and any multipath delay spread, there exists an observation interval long enough to allow linear decorrelation. For a fixed delay spread and observation interval, (12) determines the maximum number of users that a GLD can handle. Similar to the asynchronous case, a GLD exists for any $M \geq 1$ whenever $K \leq N/2$, regardless of the amount of multipath delay spread.

3.3 Asynchronous users, multipath channels

Finally, consider asynchronous uncoded BPSK transmission through multipath channels. Because of asynchronism, the path delays of all users are arbitrary. Let the observation interval span exactly M complete received bits, such that $N_c = (M + L - 1)N$. The following limitations of a GLD can be inferred in this case.

$$K \leq N - \frac{NL - 1}{M + 2L - 1}, \tag{14}$$

or, equivalently,

$$K < N \quad \text{and} \quad M \geq \frac{K(2L-1) - N(L-1) - 1}{N - K}. \tag{15}$$

The above results closely parallel those from Section 3.1 and Section 3.2. The case of $M = 1$ is of special interest since it is the usual way in which linear multiuser detectors are implemented. When $M = 1$, (14) implies that the maximum number of users that a GLD can handle is

$$K_{\max} = \begin{cases} \lfloor \frac{N}{2} \rfloor, & L > 1 \\ \lceil \frac{N}{2} \rceil, & L = 1. \end{cases} \tag{16}$$

Therefore, when a one-shot GLD is used, multipath propagation and/or asynchronism decreases the maximum number of users by a factor of two. Also, for any observation interval, as the multipath delay spread increases, the maximum number of users approaches $N/2$. The minimum observation window length M is plotted for different numbers of users K in Figure 2, using $N = 100$ and $L = 2$ in (14). The figure shows that the minimum required window length is equal to one bit period for $K \leq N/2$, and then increases rapidly to a final length of $L(N-2)$ bit periods.

The analyses in Sections 3.1, 3.2 and 3.3 indicate that if a GLD does not exist for $T = (M/N - L + 1)T_c$, then there is no observation interval for which a GLD exists.

4 Effect of Coding or Block Modulation

A drawback of conventional linear multiuser detection is the coding-spreading conflict (see, e.g. [11]). If, for instance, a rate $R < 1$ channel code is used at the transmitter to convert information bits to encoded bits prior to spreading, the maximum number of users for which a conventional linear decorrelator exists is reduced to NR, where N is the spreading gain over information bits. Therefore, any allocation of bandwidth expansion in coding reduces the interference suppression capability of linear decorrelators by the same factor. This loss of linear multiuser detector capability in the presence of coding is due to the fact that conventional linear multiuser detectors do not utilize the knowledge of channel codes. Instead, they treat the encoded signals as the signals to be detected, thereby losing the spreading gain that is used up by coding. The same is true when multi-dimensional modulation is used, that is, $m > 1$ information bits are modulated to one of 2^m symbol waveforms that occupy more than a single dimension

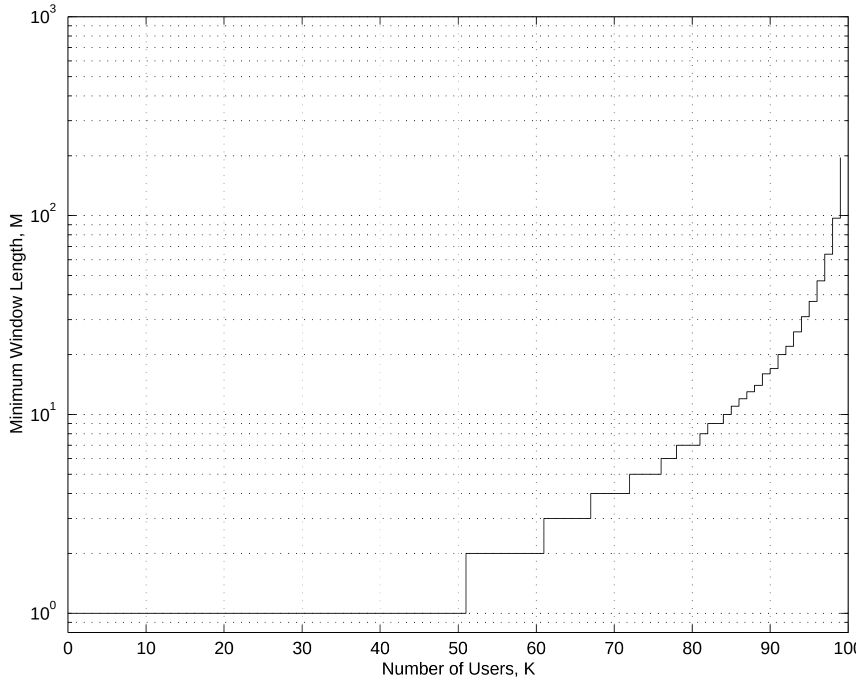

Figure 2: Minimum observation interval versus number of users for $N = 100$ and $L = 2$.

in the signal space. An important example is M-ary orthogonal modulation, in which $\log_2 M$ information bits are block encoded to one of M orthogonal codewords, each of length M. This scheme is used in the uplink of the IS-95 CDMA cellular standard. The use of conventional linear decorrelators for M-ary orthogonal modulated CDMA systems has been suggested in literature (see, e.g., [12]). Since the code rate in the case of M-ary orthogonal modulation is $(\log_2 M)/M$, the number of users that a linear decorrelator can handle is decreased to $(N \log_2 M)/M$, which, for $M = 64$ and $N = 128/3$ (in IS-95; N is over the input bitstream to the M-ary modulator), is merely 4. Evidently, the capability of conventional linear detection is severely limited in this case.

A pertinent question is to see if knowledge of channel codes or modulation scheme can improve the interference suppression capability of linear detectors. This question is addressed here in the framework developed in Section 2. Consider an encoder that maps m information bits to one codeword, which is then multiplied by the spreading code. Let $\{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_{2^m}\}$ be the set of 2^m desired user's codewords as seen at the output of the chip-matched filter. Without loss of generality, we can assume that the observation interval spans the length of exactly one codeword. If the observation interval spans more than one codeword length, the codebook can be redefined to include all possible strings of codewords in the observation interval. The desired signal space $\mathcal{S}_D$ is the space spanned by the above set of code vectors. Its dimension, N_D, determines the interference suppression capability of GLD's, according to the condition in (7). Therefore, knowledge of the space spanned by all codewords can be utilized in designing linear receivers that can potentially overcome some of the limitations of conventional linear multiuser detectors. If all the K users are synchronous, then a GLD exists if, and only if,

$$K < \frac{Nm}{N_D} \tag{17}$$

An interesting observation can be made from (17). If a single dimensional modulation scheme[4] such as PAM, M-PSK or QAM is used to convert m information bits to one symbol, then $N_D = 1$. Consequently, the number of users that a GLD can handle is constrained by $K < mN$, an increase compared to the case of BPSK modulation by a factor of m.

Consider a binary code with code rate $R < 1$ that maps m information bits to $M = m/R$ encoded bits, which are then BPSK modulated. Define an $M \times 2^m$ codebook matrix with each code symbol as a column. The linear span of the columns of this matrix is the signal subspace $\mathcal{S}_D$, with dimension N_D. If the codebook is rank deficient ($N_D < M$), then a GLD permits $K_{\max} = \lfloor mN/N_D \rfloor$ users, which is higher than the $K_{\max} = \lfloor mN/M \rfloor$ achievable with a conventional decorrelator. If the codebook is full rank ($N_D = M$), however, the conventional linear decorrelator that uses no information about the code is optimal in terms of maximum allowable users. Such is the case with M-ary orthogonal modulation, in which the codebook has full rank due to orthogonality of all M codewords. Therefore, $K < (N \log_2 M)/M$ is a fundamental limitation of linear reception in M-ary orthogonal modulated systems. Written in terms of m, the number of bits per M-ary symbol, the above constraint is equivalent to

$$K < \frac{Nm}{2^m}. \tag{18}$$

Therefore, when M-ary orthogonal modulation is used, the interference suppression capability of linear detectors decreases exponentially with increase in block length.

5 Conclusions

This paper showed that though linear multiuser detection is possible in a wide range of scenarios where linear receivers are generally not believed to be applicable, its ability to suppress interference is constrained by the dimensions of spaces spanned by desired and interfering signals. Limitations of linear detection in certain practically important scenarios were studied in a general framework for linear detection derived in this paper. This theory can also be applied to investigate the limitations of linear detectors or to design linear decorrelators in many other situations not addressed in this paper (e.g. presence of coding *and* multipath propagation).

The dimensional limitations of linear multiuser detection, along with their requirement of short codes[5], suggests that nonlinear multiuser detection schemes could be attractive alternatives. We know that there exists at least one nonlinear multiuser detector—the optimal ML detector—that is not affected by the limitations of linear detection schemes discussed in this paper. One attractive solution is an optimal iterative interference canceler [13, 14] that is shown in [14] to approximate minimum probability of error multiuser detection with computational complexity of the order of a conventional parallel interference canceler. Further investigation is required in the analysis of suboptimal nonlinear receivers in various scenarios and performance comparison of linear and nonlinear multiuser detectors.

[4]Also called "linear" modulation. Quadrature modulated signals such as QAM also have a single dimension over the complex field.

[5]Neither the current IS-95 CDMA standard nor any of the proposals for wideband CDMA for third generation wireless standards support short codes.

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