Math 30820: Honors Algebra IV
Problem Set 8

1. Artin, Chapter 16, Problem 1.1.

2. Let K be a field and let $f(x) \in K[x]$ be a monic degree n polynomial. Assume that $K \subset L$ is a field extension such that L contains all the roots $\alpha_1, \ldots, \alpha_n$ of f, counted with multiplicity. Prove that the discriminant of $f(x)$ satisfies the identity
$$\Delta(f) = (-1)^{\binom{n}{2}} \prod_{i=1}^{n} f'(\alpha_i).$$

3. Let $f(x) = x^5 + px + q$, with p, q elements of a field K. Prove that the discriminant of f satisfies $\Delta(f) = 5^5 q^4 + 4^4 p^5$.

4. Say that a field extension $K \subset L$ is a *normal extension* if all irreducible polynomials $f(x) \in K[x]$ which have a root in L split into linear factors in L. If $K \subset L$ is finite, we proved in class that this holds if and only if $K \subset L$ is the splitting field of some polynomial. Let $\alpha \in \mathbb{R}$ satisfy $\alpha^4 = 5$. **Question**: Are the following extensions normal or not?
   (a) $\mathbb{Q} \subset \mathbb{Q}(i\alpha^2)$
   (b) $\mathbb{Q}(i\alpha^2) \subset \mathbb{Q}(\alpha + i\alpha)$. You should also justify that this is indeed a field extension.
   (c) $\mathbb{Q} \subset \mathbb{Q}(\alpha + i\alpha)$.

5. Artin, Chapter 16, Problem 3.2

6. Artin, Chapter 16, Problem 4.1