Einstein's Physical Strategy in Pursuit of General Relativity, Particle Physicists' Approach to Gravitation, and Physical Relativity

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Although Einstein is widely believed to have found the field equations of General Relativity using a mathematical-philosophical strategy, recent Einstein scholarship has called attention to his use of a two-pronged strategy in his process of pursuit. The mathematical-philosophical approach involved various principles of relativity, covariance, equivalence, Mach and the like. However, Einstein also pursued a more physical strategy involving an analogy to Maxwell's electromagnetism and less conjectural, more nuts-and-bolts physical principles involving the energy-momentum complex as a source term and the need for its conservation [Klein et al., 1995; Schulmann et al., 1998; Janssen, 2005; Brading, 2005]. Poincaré's notions of the conventionality of geometry and of a universal force that would make physical laws formulated in one geometry simulate another geometry [Poincaré, 1902] could have been given a serious physical exemplification if Einstein's pursuit of the physical strategy had succeeded. As it happened, Einstein ultimately concluded that the mathematical-philosophical strategy led to success and the physical strategy led to failure, so he permanently turned against the latter and its history was largely forgotten. For Einstein the empirical geometry was the right geometry, because then the combination of physics and mathematics, not the mathematics alone, was simplest. His conclusions were followed by the community of general relativists and by most philosophers and historians of science. For example, Carnap singled out for special praise Reichenbach's stipulation that universal forces be chosen to vanish [Reichenbach, 1928, p. vii].

Meanwhile, developments involving group theory in quantum mechanics, such as by Wigner, called for classifying all possible interactions of fields in terms of the Lorentz group with various masses and various spins. In the late 1930s Pauli and Fierz found that the theory of a non-interacting massless spin 2 field in Minkowski space-time was just the linear approximation of Einstein's GTR [Pauli and Fierz, 1939; Fierz and Pauli, 1939]. From that time a large number of physicists, many of them eminent and many of them particle physicists, in effect revived, developed and extended Einstein's physical strategy, though generally not under that description. Einstein's equations are remarkably nonlinear and complex when construed as a field theory of a symmetric rank 2 tensor gravitational potential in flat space-time, yet so simple and inevitable using the mathematics of Riemannian geometry. Could they really be *derived* using such mundane particle physics principles as Lorentz covariance, elimination of negative-energy degrees of freedom to secure stability and reasonable thermodynamic behavior, *etc.*? Contributions to this physical strategy were occasionally unwitting, as in the Birkhoff-Weyl exchange [Weyl, 1944]: Birkhoff proposed a non-Einsteinian theory in flat space-time on grounds of naive mathematical simplicity, whereas Weyl showed on physical grounds that Birkhoff's project could not succeed while differing from the linearized Einstein equations.

Differential geometry also advanced. While metric-based Riemannian geometry was the state of the art in 1915 when Einstein settled on his famous and highly successful equations, in the late 1910s the connection would be discovered as an independent geometric object. These results deprived Riemannian geometry of any claim of being the end of mathematical history, while metric-affine and purely affine geometries have not been especially fruitful, though the connection would eventually provide a key analogy for gauge theories. In the 1920s Levi-Civita first conceived of the generic possibility of two metrics on the same space [Levi-Civita, 1926], without the restriction of their being conformally related. Given conformal relatedness, the Einstein-Fokker geometric formulation of Nordström's scalar gravity and perhaps some massive variants [Freund and Nambu, 1968; Pitts, 2009] would be conceivable, but not a bimetric formulation of Einstein's equations or any other tensor theory of gravity.

By 1940 Rosen, with bimetric geometry in view, had called for attempting to derive Einstein's equations on a flat background from basic principles [Rosen, 1940]. In the early 1950s Einstein remained deeply skeptical of this revival of his own earlier physical strategy [Feynman et al., 1995]; "...it would be practically impossible for anybody to hit on the gravitational equations" without the "exceedingly strong restrictions on the theoretical possibilities" imposed by "the principle of general relativity" [Einstein, 1954]. But Einstein's equations indeed were derived on plausible particle physics principles in flat space-time by 1955 [Kraichnan, 1955; Gupta, 1954, 1957; Thirring, 1961; Huggins, 1962; Feynman et al., 1995; Weinberg, 1964a,b; Ogievetsky and Polubarinov, 1965; Nachtmann et al., 1968; van Nieuwenhuizen, 1973; Deser, 1970; Boulware and Deser, 1975; Fang and Fronsdal, 1979; Cavalleri and Spinelli, 1980; Davies and Fang, 1982; Grishchuk et al., 1984; Pitts and Schieve, 2001; Boulanger and Esole, 2002]. Subsequent developments of the physical strategy would include supergravity [Boulware et al., 1979], in which a local supersymmetry relates spin 2 gravitons and spin $\frac{3}{2}$ fermions. By the turn of the millennium Boulanger and Esole could express a generally held view among particle physicists: "[i]t is well appreciated that general relativity is the unique way to consistently deform the Pauli-Fierz action $\int \mathcal{L}_2$ for a free massless spin-2 field under the assumption of locality, Poincaré invariance, preservation of the number of gauge symmetries and the number of derivatives in $\int \mathcal{L}_2$" [*sic*] [Boulanger and Esole, 2002]. The original flat metric merges with the gravitational potential so that only an effective curved metric appears in the field equations. This coalescence happens for basic particle physics reasons such as Lorentz covariance, avoidance negative energy degrees of freedom, coupling to the only available source term, and the existence of an action principle.

Despite the remarkable success of the revival of Einstein's physical strategy, some authors remained skeptical. Is not the disappearance of the flat background metric from the final result, rather than being a stunning success, instead a stunning failure for the use of a flat metric, a good reason to deny its existence and even its instrumental utility? Perhaps, but perhaps not; its role in the context of discovery and its utility in expressing Einstein's equations in a form more like other forces might count for something. Neither should one assume that the whole of the physics is in the field equations, when notions such as topology, boundary conditions, and possible inequalities bounding the field variables might reflect the background metric. So might the need to define equal times for quantization with equal-time commutation relations.

Einstein's mathematical-philosophical approach is also perceived by some to have the imprimatur of historical progress, and hence to be preferable to the physical strategy, even when the success of the latter is granted [Ehlers, 1973, pp. 84, 85]. Besides the selective history required to make a result from 1915 more advanced than mid-century work, this view is subject to the danger of enshrining historical accidents, such as Einstein's own failure in pursuing the physical strategy, as manifestations of inevitable historical progress. Lakatos, by contrast, has urged that one must rationally reconstruct history in order to assess scientific progress. Thus one cannot assess progress in 20th century space-time theory without due attention to particle physicists' success in pursuing Einstein's physical strategy. That success might be seen as a way to view Einstein's equations as describing a constructive rather than principle theory.

It is therefore appropriate to pursue a more physical approach to the philosophy of space-time theory. This approach would involve thorough and current attention to the physics literature, as opposed to selectively focusing on developments of lines of thought that were mature in 1915. Harvey Brown, in proposing a more physical approach to relativistic physics, has called attention to theories in which merely listing the geometric objects present in the theory gives little insight into the theory's phenomenology [Brown, 2005]. There are, one might notice, theories with multiple metrics, such as scalar-tensor theories [Weinstein, 1996] and massive variants of Nordström's scalar gravity [Freund and Nambu, 1968; Pitts, 2009].

There are possible theories in which different matter fields see different metrics, or in which only some matter fields see a preferred reference frame. There need not be any one true metric, or even one true geometry, for a physical theory. Given such rich possibilities, it is unhelpful to assume for each physical theory that there is exactly one type of space-time that it instantiates. It is more adequate to focus attention on the theory's field equations or Lagrangian density, which need not admit any useful briefer summary. That General Relativity admits a brief summary in terms of Riemannian geometry is thus seen to be an important special property of that theory.

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