Home-work #5

Problems: 6.7; 6.10; 6.11; 6.12; 6.16; 6.19; 6.24; 6.30; 6.33; 6.38; 6.41; 6.45; 6.46; 6.47; 6.52.

6.7 (a) This portion of the problem calls for a determination of the maximum load that can be applied without plastic deformation (F_y). Taking the yield strength to be 345 MPa, and employment of Equation 6.1 leads to

$$F_y = \sigma_y A_0 = (345 \text{ x } 10^6 \text{ N/m}^2)(130 \text{ x } 10^{-6} \text{ m}^2)$$

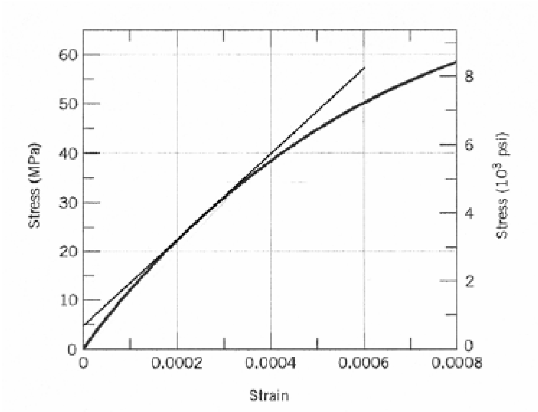
$$= 44{,}850 \text{ N} \quad (10{,}000 \text{ lb}_f)$$

(b) The maximum length to which the sample may be deformed without plastic deformation is determined from Equations 6.2 and 6.5 as

$$l_i = l_0\left(1 + \frac{\sigma}{E}\right)$$

$$= (76 \text{ mm})\left[1 + \frac{345 \text{ MPa}}{103 \text{ x } 10^3 \text{ MPa}}\right] = 76.25 \text{ mm} \quad (3.01 \text{ in.})$$

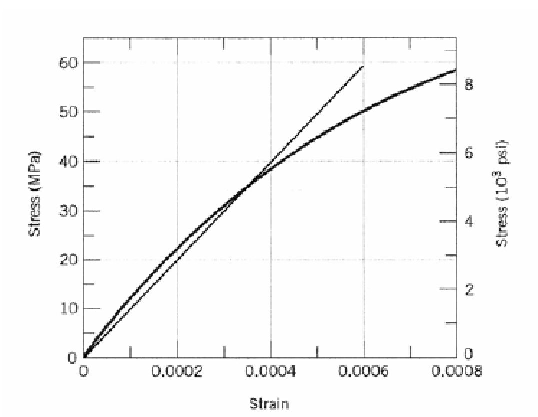
6.10 (a) This portion of the problem asks that the tangent modulus be determined for the gray cast iron, the stress-strain behavior of which is shown in Figure 6.22. In the figure below is shown a tangent draw on the curve at a stress of 25 MPa.



The slope of this line (i.e., $\Delta\sigma/\Delta\varepsilon$), the tangent modulus, is computed as follows:

$$\frac{\Delta\sigma}{\Delta\varepsilon} = \frac{57 \text{ MPs} - 0 \text{ MPa}}{0.0006 - 0} = 95{,}000 \text{ MPa} = 95 \text{ GPa} \ \ (13.8 \times 10^6 \text{ psi})$$

(b) The secant modulus taken from the origin is calculated by taking the slope of a secant drawn from the origin through the stress-strain curve at 35 MPa (5,000 psi). This secant modulus is drawn on the curve shown below:



The slope of this line (i.e., $\Delta\sigma/\Delta\varepsilon$), the secant modulus, is computed as follows:

$$\frac{\Delta\sigma}{\Delta\varepsilon} = \frac{60 \text{ MPs} - 0 \text{ MPa}}{0.0006 - 0} = 100{,}000 \text{ MPa} = 100 \text{ GPa} \ (14.5 \text{ x } 10^6 \text{ psi})$$

6.11 We are asked, using the equation given in the problem statement, to verify that the modulus of elasticity values along [110] directions given in Table 3.3 for aluminum, copper, and iron are correct. The α, β, and γ parameters in the equation correspond, respectively, to the cosines of the angles between the [110] direction and [100], [010] and [001] directions. Since these angles are 45°, 45°, and 90°, the values of α, β, and γ are 0.707, 0.707, and 0, respectively. Thus, the given equation takes the form

$$\frac{1}{E_{<110>}}$$

$$= \frac{1}{E_{<100>}} - 3\left(\frac{1}{E_{<100>}} - \frac{1}{E_{<111>}}\right)\left[(0.707)^2(0.707)^2 + (0.707)^2(0)^2 + (0)^2(0.707)^2\right]$$

$$= \frac{1}{E_{<100>}} - (0.75)\left(\frac{1}{E_{<100>}} - \frac{1}{E_{<111>}}\right)$$

Utilizing the values of $E_{<100>}$ and $E_{<111>}$ from Table 3.3 for Al

$$\frac{1}{E_{<110>}} = \frac{1}{63.7 \text{ GPa}} - (0.75)\left[\frac{1}{63.7 \text{ GPa}} - \frac{1}{76.1 \text{ GPa}}\right]$$

Which leads to, $E_{<110>}$ = 72.6 GPa, the value cited in the table.

For Cu,

$$\frac{1}{E_{<110>}} = \frac{1}{66.7 \text{ GPa}} - (0.75)\left[\frac{1}{66.7 \text{ GPa}} - \frac{1}{191.1 \text{ GPa}}\right]$$

Thus, $E_{<110>}$ = 130.3 GPa, which is also the value cited in the table.

Similarly, for Fe

$$\frac{1}{E_{<110>}} = \frac{1}{125.0 \text{ GPa}} - (0.75)\left[\frac{1}{125.0 \text{ GPa}} - \frac{1}{272.7 \text{ GPa}}\right]$$

And $E_{<110>}$ = 210.5 GPa, which is also the value given in the table.

6.12 This problem asks that we derive an expression for the dependence of the modulus of elasticity, E, on the parameters A, B, and n in Equation 6.25. It is first necessary to take dE_N/dr in order to obtain an expression for the force F; this is accomplished as follows:

$$F = \frac{dE_N}{dr} = \frac{d\left(-\frac{A}{r}\right)}{dr} + \frac{d\left(\frac{B}{r^n}\right)}{dr}$$

$$= \frac{A}{r^2} - \frac{nB}{r^{(n+1)}}$$

The second step is to set this dE_N/dr expression equal to zero and then solve for r (= r_0). The algebra for this procedure is carried out in Problem 2.14, with the result that

$$r_0 = \left(\frac{A}{nB}\right)^{1/(1-n)}$$

Next it becomes necessary to take the derivative of the force (dF/dr), which is accomplished as follows:

$$\frac{dF}{dr} = \frac{d\left(\frac{A}{r^2}\right)}{dr} + \frac{d\left(-\frac{nB}{r^{(n+1)}}\right)}{dr}$$

$$= -\frac{2A}{r^3} + \frac{(n)(n+1)B}{r^{(n+2)}}$$

Now, substitution of the above expression for r_0 into this equation yields

$$\left(\frac{dF}{dr}\right)_{r_0} = -\frac{2A}{\left(\frac{A}{nB}\right)^{3/(1-n)}} + \frac{(n)(n+1)B}{\left(\frac{A}{nB}\right)^{(n+2)/(1-n)}}$$

which is the expression to which the modulus of elasticity is proportional.

6.16 This problem asks that we compute Poisson's ratio for the metal alloy. From Equations 6.5 and 6.1

$$\varepsilon_z = \frac{\sigma}{E} = \frac{F}{A_0 E} = \frac{F}{\pi\left(\frac{d_0}{2}\right)^2 E} = \frac{4F}{\pi d_0^2 E}$$

Since the transverse strain ε_x is just

$$\varepsilon_x = \frac{\Delta d}{d_0}$$

and Poisson's ratio is defined by Equation 6.8, then

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\Delta d / d_0}{\left(\frac{4F}{\pi d_0^2 E}\right)} = -\frac{d_0 \Delta d \, \pi E}{4F}$$

$$= -\frac{(10 \text{ x } 10^{-3} \text{ m})(-7 \text{ x } 10^{-6} \text{ m})\,(\pi)(100 \text{ x } 10^{9} \text{ N/m}^2)}{(4)(15{,}000 \text{ N})} = 0.367$$

6.19 We are asked to ascertain whether or not it is possible to compute, for brass, the magnitude of the load necessary to produce an elongation of 1.9 mm (0.075 in.). It is first necessary to compute the strain at yielding from the yield strength and the elastic modulus, and then the strain experienced by the test specimen. Then, if

$$\varepsilon(\text{test}) < \varepsilon(\text{yield})$$

deformation is elastic, and the load may be computed using Equations 6.1 and 6.5. However, if

$$\varepsilon(\text{test}) > \varepsilon(\text{yield})$$

computation of the load is not possible inasmuch as deformation is plastic and we have neither a stress-strain plot nor a mathematical expression relating plastic stress and strain. We compute these two strain values as

$$\varepsilon(\text{test}) = \frac{\Delta l}{l_0} = \frac{1.9 \text{ mm}}{380 \text{ mm}} = 0.005$$

and

$$\varepsilon(\text{yield}) = \frac{\sigma_y}{E} = \frac{240 \text{ MPa}}{110 \text{ x } 10^3 \text{ MPa}} = 0.0022$$

Therefore, computation of the load is *not possible* since ε(test) > ε(yield).

6.24 Using the stress-strain plot for a steel alloy (Figure 6.21), we are asked to determine several of its mechanical characteristics.

(a) The elastic modulus is just the slope of the initial linear portion of the curve; or, from the inset and using Equation 6.10

$$E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} = \frac{(1300 - 0)\ \text{MPa}}{\left(6.25 \times 10^{-3} - 0\right)} = 210 \times 10^3\ \text{MPa} = 210\ \text{GPa}\ \ (30.5 \times 10^6\ \text{psi})$$

The value given in Table 6.1 is 207 GPa.

(b) The proportional limit is the stress level at which linearity of the stress-strain curve ends, which is approximately 1370 MPa (200,000 psi).

(c) The 0.002 strain offset line intersects the stress-strain curve at approximately 1570 MPa (228,000 psi).

(d) The tensile strength (the maximum on the curve) is approximately 1970 MPa (285,000 psi).

6.30 This problem calls for the computation of ductility in both percent reduction in area and percent elongation. Percent reduction in area is computed using Equation 6.12 as

$$\%\text{RA} = \frac{\pi\left(\frac{d_0}{2}\right)^2 - \pi\left(\frac{d_f}{2}\right)^2}{\pi\left(\frac{d_0}{2}\right)^2} \times 100$$

in which d_0 and d_f are, respectively, the original and fracture cross-sectional areas. Thus,

$$\%\text{RA} = \frac{\pi\left(\frac{12.8\ \text{mm}}{2}\right)^2 - \pi\left(\frac{8.13\ \text{mm}}{2}\right)^2}{\pi\left(\frac{12.8\ \text{mm}}{2}\right)^2} \times 100 = 60\%$$

While, for percent elongation, we use Equation 6.11 as

$$\%\text{EL} = \left(\frac{l_f - l_0}{l_0}\right) \times 100$$

$$= \frac{74.17\ \text{mm} - 50.80\ \text{mm}}{50.80\ \text{mm}} \times 100 = 46\%$$

6.33 The modulus of resilience, yield strength, and elastic modulus of elasticity are related to one another through Equation 6.14; the value of E for steel given in Table 6.1 is 207 GPa. Solving for σ_y from this expression yields

$$\sigma_y = \sqrt{2U_r E} = \sqrt{(2)(2.07 \text{ MPa})(207 \text{ x } 10^3 \text{ MPa})}$$

$$= 925 \text{ MPa} \ (134{,}000 \text{ psi})$$

6.38 We are asked to compute how much elongation a metal specimen will experience when a true stress of 415 MPa is applied, given the value of n and that a given true stress produces a specific true strain. Solution of this problem requires that we utilize Equation 6.19. It is first necessary to solve for K from the given true stress and strain. Rearrangement of this equation yields

$$K = \frac{\sigma_T}{(\varepsilon_T)^n} = \frac{345 \text{ MPa}}{(0.02)^{0.22}} = 816 \text{ MPa} \ (118{,}000 \text{ psi})$$

Next we must solve for the true strain produced when a true stress of 415 MPa is applied, also using Equation 6.19. Thus

$$\varepsilon_T = \left(\frac{\sigma_T}{K}\right)^{1/n} = \left(\frac{415 \text{ MPa}}{816 \text{ MPa}}\right)^{1/0.22} = 0.0463 = \ln\left(\frac{l_i}{l_0}\right)$$

Now, solving for l_i gives

$$l_i = l_0 e^{0.0463} = (500 \text{ mm}) e^{0.0463} = 523.7 \text{ mm} \ (20.948 \text{ in.})$$

And finally, the elongation Δl is just

$$\Delta l = l_i - l_0 = 523.7 \text{ mm} - 500 \text{ mm} = 23.7 \text{ mm} \ (0.948 \text{ in.})$$

6.41 This problem calls for us to compute the toughness (or energy to cause fracture). The easiest way to do this is to integrate both elastic and plastic regions, and then add them together.

$$\text{Toughness} = \int \sigma \, d\varepsilon$$

$$= \int_0^{0.007} E\varepsilon \, d\varepsilon + \int_{0.007}^{0.60} K\varepsilon^n \, d\varepsilon$$

$$= \frac{E\varepsilon^2}{2}\Bigg|_0^{0.007} + \frac{K}{(n+1)}\varepsilon^{(n+1)}\Bigg|_{0.007}^{0.60}$$

$$= \frac{103 \text{ x } 10^9 \text{ N/m}^2}{2}(0.007)^2 + \frac{1520 \text{ x } 10^6 \text{ N/m}^2}{(1.0 + 0.15)}\left[(0.60)^{1.15} - (0.007)^{1.15}\right]$$

$$= 7.33 \text{ x } 10^8 \text{ J/m}^3 \;\; (1.07 \text{ x } 10^5 \text{ in.-lb}_f\text{/in.}^3)$$

6.45 (a) We are asked to determine both the elastic and plastic strain values when a tensile force of 110,000 N (25,000 lb_f) is applied to the steel specimen and then released. First it becomes necessary to determine the applied stress using Equation 6.1; thus

$$\sigma = \frac{F}{A_0} = \frac{F}{b_0 d_0}$$

where b_0 and d_0 are cross-sectional width and depth (19 mm and 3.2 mm, respectively). Thus

$$\sigma = \frac{110,000 \text{ N}}{(19 \times 10^{-3}\text{ m})(3.2 \times 10^{-3}\text{ m})} = 1.810 \times 10^{9} \text{ N/m}^2 = 1810 \text{ MPa} \quad (265,000 \text{ psi})$$

From Figure 6.21, this point is in the plastic region so the specimen will be both elastic and plastic strains. The total strain at this point, ε_t, is about 0.020. We are able to estimate the amount of permanent strain recovery ε_e from Hooke's law, Equation 6.5 as

$$\varepsilon_e = \frac{\sigma}{E}$$

And, since E = 207 GPa for steel (Table 6.1)

$$\varepsilon_e = \frac{1810 \text{ MPa}}{207 \times 10^3 \text{MPa}} = 0.0087$$

The value of the plastic strain, ε_p is just the difference between the total and elastic strains; that is

$$\varepsilon_p = \varepsilon_t - \varepsilon_e = 0.020 - 0.0087 = 0.0113$$

(b) If the initial length is 610 mm (24.0 in.) then the final specimen length l_i may be determined from a rearranged form of Equation 6.2 using the plastic strain value as

$$l_i = l_0(1 + \varepsilon_p) = (610 \text{ mm})(1 + 0.0113) = 616.7 \text{ mm (24.26 in.)}$$

6.46 (a) We are asked to compute the Brinell hardness for the given indentation. It is necessary to use the equation in Table 6.5 for HB, where P = 1000 kg, d = 2.50 mm, and D = 10 mm. Thus, the Brinell hardness is computed as

$$\text{HB} = \frac{2P}{\pi D\left[D - \sqrt{D^2 - d^2}\right]}$$

$$= \frac{(2)(1000 \text{ kg})}{(\pi)(10 \text{ mm})\left[10 \text{ mm} - \sqrt{(10 \text{ mm})^2 - (2.50 \text{ mm})^2}\right]} = 200.5$$

(b) This part of the problem calls for us to determine the indentation diameter d which will yield a 300 HB when P = 500 kg. Solving for d from the equation in Table 6.5 gives

$$d = \sqrt{D^2 - \left[D - \frac{2P}{(\text{HB})\pi D}\right]^2}$$

$$= \sqrt{(10 \text{ mm})^2 - \left[10 \text{ mm} - \frac{(2)(500 \text{ kg})}{(300)(\pi)(10 \text{ mm})}\right]^2} = 1.45 \text{ mm}$$

6.47 This problem calls for estimations of Brinell and Rockwell hardnesses.

(a) For the brass specimen, the stress-strain behavior for which is shown in Figure 6.12, the tensile strength is 450 MPa (65,000 psi). From Figure 6.19, the hardness for brass corresponding to this tensile strength is about 125 HB or 70 HRB.

(b) The steel alloy (Figure 6.21) has a tensile strength of about 1970 MPa (285,000 psi) [Problem 6.24(d)]. This corresponds to a hardness of about 560 HB or ~55 HRC from the line (extended) for steels in Figure 6.19.

6.52 The working stresses for the two alloys the stress-strain behaviors of which are shown in Figures 6.12 and 6.21 are calculated by dividing the yield strength by a factor of safety, which we will take to be 2. For the brass alloy (Figure 6.12), since σ_y = 250 MPa (36,000 psi), the working stress is 125 MPa (18,000 psi), whereas for the steel alloy (Figure 6.21), σ_y = 1570 MPa (228,000 psi), and, therefore, σ_w = 785 MPa (114,000 psi).