Cultural Dissemination using a Quantum Model

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Abstract

Axelrod's [Axelrod, 1997] cultural dissemination model introduces an agent-based simulation where random agent interactions transmit culture through an agent population, and the system evolves over time to form multiple stable homogeneous cultural regions. We expand upon this work by introducing a quantum model. Agents are represented by quantum registers, and agent interactions are quantum operations performed on those registers. Results indicate that multiple stable *heterogeneous* cultural regions form as the population evolves, the number of regions formed is greater in the quantum model, and there is a greater diversity in the sizes of the cultural regions.

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Cultural Dissemination using a Quantum Model

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Quantum computation and quantum information have become active research areas in recent years. With the discovery of a quantum algorithm for factoring prime numbers by Shor [Shor, 1994] and quantum searching of databases by Grover [Grover, 1996] which both outperform classical algorithms, much focus has been placed on finding other quantum algorithms and application areas which can improve upon their classical counterpart. We have taken a different approach in that we are using quantum computation for simulation of a computational social model. Here our focus is on the agents, their interactions, and the social structures and behaviors that emerge from the agent population over time. We express the social theory in a computational quantum model by using the principles of quantum computation such as qubits, superposition of states, probabilistic measurement, and quantum operations. We investigate the process of transforming a discrete computational social model into a quantum model in a way that the underlying system maintains a conceptual link with the social theory. In particular, we explore how culture as a property of humans evolves over time as that person interacts with humans of other cultures. Cultural features can disseminate through the population, reaching remote people with no prior knowledge of that culture, purely by local interactions among the individuals in the population. Also, various cultures present within the initial population die out to be replaced by new hybrid cultures, and common cultural regions form where local individuals in these regions share common aspects of the same culture. For the computational social model of culture and its dissemination through a population, we use Axelrod's [Axelrod, 1997] agent-based simulation as a starting point. It displays characteristics of the social theory that is appealing, particularly the formation of common cultural regions.

1 Quantum Model

When converting a discrete model to a quantum model, it is important to investigate all aspects of the computation involved as well as the data representations. It is for this reason that we picked a relatively simple computation social model as a starting point because the data structures and calculations are limited in scope. It allows us to focus on exactly what it means for a discrete agent to become a quantum agent, and at the same time it kept the quantum calculations relatively simple. This is crucial because complex quantum systems can quickly become infeasible to simulate on a classical computer, and we wanted to perform actual simulations for comparison against the discrete model.

The three primary areas of the discrete model that we needed to focus on to transform it to a quantum model are the data structures for the agents (culture), the calculation of the cultural similarity (probability of interaction), and the agent interactions (cultural dissemination). The following sections describes these three parts of the model transformation in more detail.

1.1 Agent Representation

The discrete model represents an agent or culture as composed of five individual features where each feature can assume one of ten trait values. We translated this into the quantum model with each agent being represented by five 3-qubit quantum registers. A 3-qubit register provides us with eight quantum states for the trait values which is less than the ten trait values in the discrete model, so here is one place where we differ slightly from the discrete model. We could have gone with 4-qubit registers and ignored six of the quantum states by setting their amplitudes to 0; however, if the quantum model portrayed the same behavior as the discrete model then a slightly smaller number of trait values should just correspond to a smaller average number of cultural regions. All told, a single agent is represented by 40 complex numbers; and with one hundred agents on a 10 by 10 grid, we have a total of 4000 complex numbers for the simulation.

The discrete model treats the features as being independent of each other; while all of the features are used to determine the cultural similarity, when agents interact and cultural dissemination occurs then the trait value for a feature only transfers to that same feature for the active agent. For this reason, we can consider the quantum registers as also being independent. An alternate representation would be a single 15-qubit register for the culture; while this has some merits in that one can consider that cultural features are not truly independent of each other and this representation would allow those features to interact with each other, we decided to stick with the original design. Even if we did want to have a 15-qubit register, this would mean maintaining 2^{15} complex numbers for each agent which quickly becomes infeasible to simulate for even modest sized agent populations.

1.2 Cultural Similarity

An important calculation in the model is the cultural similarity; with the discrete model, a straight comparison of the trait values for each feature between two agents provides a simple metric. However in a quantum system, we cannot directly compare two quantum registers; we need some sort of well-defined metric. There are two metrics used widely within the quantum information community [Nielsen and Chuang, 2000]; one is the trace distance and the other is the fidelity. Of the two, only the trace distance can be considered a true distance metric as it is symmetric and satisfies the triangle inequality; so for two quantum registers ψ and ϕ, the trace distance between them is:

$$D(\psi, \phi) = \frac{1}{2}\, tr|(\psi - \phi)|$$

where $|A|$ is defined as $\sqrt{A^\dagger A}$ and $A^\dagger$ is the Hermitian conjugate of the matrix A and tr is the trace (sum of the diagonal elements) of the matrix. The trace distance has a close relationship to the Kolmogorov distance used in classical probability theory for determining how similar two probability distributions are to one another. In fact if the two quantum states commute; that is, both are represented in the same basis, then the quantum trace distance is equal to the classical trace distance:

$$D(p, q) = \frac{1}{2} \sum |p - q|$$

where p and q are probability distributions and $|a|$ is the absolute value of a. All of our quantum states commute because we are using the standard computational basis of $|0\rangle$ and $|1\rangle$ for our qubits, so the trace distance gives us a good intuitive sense of how close the two distributions are to each other.

Our culture is composed of five features where each feature is a quantum register; the trace distance only calculates the distance for one feature, so we define an average distance for all of the features to give us the cultural difference. This number is in the range $[0, 1]$ with 1 meaning 100% difference, so taking the inverse gives the cultural similarity.

$$S = 1 - \frac{1}{5} \sum_{f=1}^{5} D(\psi_f, \phi_f)$$

Identical to the discrete model, we use a random number for comparison against the cultural simularity to determine if the two agents interact.

1.3 Agent Interaction

Translating the agent interaction, the actual process of cultural dissemination, posed some difficulties. Quantum computation requires that interactions be reversible; that is, given the current state of an agent, one can undo or reverse all of the interactions to get back to the agent's initial state. However as defined in Axelrod's model, the trait value for the cultural feature of the neighbor agent overwrites the existing trait value for the active agent; this is a irreversible action because there is no way to retrieve the original value. For quantum systems, such irreversible action corresponds to a drastic loss of information in the system which is not a desirable model. Conceptually the process of cultural dissemination does not completely remove a person's knowledge of their past cultural attitudes and preferences; what seems more reasonable is that a person integrates his neighbor's culture into his own in a way that does not erase his prior culture nor blindly copies his neighbor's culture. We came up with two simple interactions; the first produces total convergence to a single culture across all of the agents; while the second produces multiple stable heterogeneous regions across the agent population.

2 Emergence of a Single Homogenous Culture

The first agent interaction we simulated is a vector addition. The motivation behind such an interaction is that when an agent is going to integrate another agent's culture into its own, the agent will find a medium point between his own culture and the culture it is adopting from the other agent. Addition of two vectors forms a new vector which is halfway between the two in terms of the angle or direction of the vector. Likewise the amplitude of the resultant vector can either increase or decrease depending upon the vectors being added.

When it has been determined that the two agents will interact and culture will transfer from the neighbor agent to the active agent, one of the features is randomly picked and a measurement is performed on the quantum register to get its state. The actual state measured is dependent upon the probability distribution as defined by the complex amplitudes of the quantum register. Here is a point where randomness enters into the calculation of the quantum model where it did not exist in the discrete model. It is certainly possible that the measurement can return a state (trait value) which does not correspond to the most likely or most probable trait for that feature.

Given that $|\psi\rangle$ is the quantum register for the neighbor agent and $|\phi\rangle$ is the quantum register for the active agent; before the interaction occurs both are in some superposition of quantum states.

$$|\psi\rangle = a_0|000\rangle + a_1|001\rangle + \cdots + a_7|111\rangle$$

$$|\phi\rangle = b_0|000\rangle + b_1|001\rangle + \cdots + b_7|111\rangle$$

A measurement performed on the quantum register $|\psi\rangle$ will return a quantum state $|\omega\rangle$ (such as $|010\rangle$) based upon a random variable and the probability distribution for that quantum register. The amplitude for that measured state $|\omega\rangle$ for $|\psi\rangle$, the neighbor agent, is then added to the amplitude for that same quantum state for $|\phi\rangle$, the active agent; thus altering the underlying probability distribution associated with the agent and its corresponding culture. Equation (1) shows the complete operation which is performed on the active agent.

$$|\phi\rangle = \frac{b_0|000\rangle + \cdots + (b_i + a_i)|\omega\rangle + \cdots + b_7|111\rangle}{\sqrt{|b_0|^2 + \cdots + |(b_i + a_i)|^2 + \cdots + |b_7|^2}} \tag{1}$$

Equation (1) shows division by a normalization factor; this is required because with the adjustment of the amplitude, the probabilities may no longer add up to 1. The normalization factor insures that they do.

The result we found for all of the runs that we performed is that the agent population converged to a single homogeneous culture. Starting from a random set of cultures for each agents, the simulation would briefly form cultural regions; but as the simulation continued, one trait value for each feature would become dominate and eventually overtake all other traits. While this behavior is interesting, it is not what we are looking for in terms of a cultural dissemination model, so we then experimented with another interaction.

3 Emergence of Multiple, Stable Heterogeneous Regions

The second agent interaction we simulated is multiplication by a scalar value. The motivation behind this interaction is similar to our motivation for vector addition but instead of adding the amplitudes together to get a new vector, we will just multiply the amplitude for the measured quantum state by a constant scalar value. This has the effect of essentially increasing the probability of that quantum state while simultaneously decreasing the probabilities of the other quantum states for that quantum register. Conceptually the interaction enforces that trait value for the feature while it downplays the other trait values.

Given that $|\psi\rangle$ is the quantum register for the neighbor agent and $|\phi\rangle$ is the quantum register for the active agent; before the interaction occurs both are in some superposition of quantum states.

$$|\psi\rangle = a_0|000\rangle + a_1|001\rangle + \cdots + a_7|111\rangle$$

$$|\phi\rangle = b_0|000\rangle + b_1|001\rangle + \cdots + b_7|111\rangle$$

Identical to the vector addition interaction, we perform a measurement on the quantum register $|\psi\rangle$ to obtain a quantum state $|\omega\rangle$. The amplitude for that measured state $|\omega\rangle$ for $|\phi\rangle$, the active agent, is then multiplied by a constant scalar value, C, to obtain a new amplitude. Equation (2) shows the complete operation which is performed on the active agent.

$$|\phi\rangle = \frac{b_0|000\rangle + \cdots + (Cb_i)|\omega\rangle + \cdots + b_7|111\rangle}{\sqrt{|b_0|^2 + \cdots + |(Cb_i)|^2 + \cdots + |b_7|^2}} \tag{2}$$

Notice that with this interaction, we do not require the amplitude a_i of the quantum state $|\omega\rangle$ from $|\psi\rangle$; this has greater plausibility as a realistic, physical operation. Equation (2) also shows the required normalization factor.

The result we get for this interaction is very much what we desired with an added bonus. Like the discrete model, the agent population converges to stable cultural regions, but the surprise is that those regions are heterogeneous and not homogeneous cultures. This means that for an individual agent, it may agree with one of its neighbors regarding the trait value for one of the features in the culture, but that agent may disagree, by having a different trait value, for another feature with that very same neighbor agent. So what we found is that the boundaries for the cultural regions are not based upon the culture as a whole but by the individual features within that culture.

A consequence of the heterogeneous regions is that there is a large amount of cultural difference throughout the agent population. With the discrete model, when the agent population converged, all of the agents either had 0% or 100% cultural difference with its neighbors. In the quantum model, many of the agents maintain various levels of cultural difference with all of its neighbors; though the similarity approaches 0% or 100% for a single feature, if you consider all of the features together as a culture then the agents have various degrees of similarity.

Multiple cultural regions may have the same trait value yet be disconnected from each other; that is, cultural regions can have the same trait value yet be disjoint groups in the agent population. Likewise not all traits survived, at least not with a high probability. This behavior is replicated across all of the features in the culture. The sizes of the cultural groups vary from just a single agent up to very large groups of agents. It is unclear what is causing these behaviors in the quantum model; it could be a special form of simulated annealing, so that is something we intend to investigate further. We do know that the scalar constant used does not appear to be a primary parameter for the model. We used a scalar constant of $C = 2$ in Equation (2) for our simulation runs, and preliminary experiments with larger and smaller values of C would indicate that the agent population still converges with the same behaviors but at a faster (for larger values) or slower rate.

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