Math 104: Finite Mathematics
Homework 3 Solutions

SECTION 5.5

21. $4!$

22. $P(6, 2)$

26. $6!$

28. $C(10, 4)$

29. $C(10, 6)$

30. Picking 4 to take along is the same as deciding which 6 to leave at home. Also, recall that $C(10, 4) = \frac{10!}{4!6!}$ while $C(10, 6) = \frac{10!}{6!4!}$ and these are the same number.

32. C(9,3) choose which 3 judges will vote the same way; the other 6 must then vote together.

43. No repetition and order matters means PERMUTATION. $P(26, 30$, to be exact. $(26 \cdot 25 \cdot 24)$

47. A hand is unordered so $C(52, 5)$.

48. To consist entirely of aces and kings means we just pick from the cards with A's or K's on them. The deck has 4 aces and 4 kings, so we have a total of 8 cards. We want 5. Thus, $C(8, 5)$.

49. There are 13 clubs in the deck. Select a hand just from these. $C(13, 5)$.

50. There are 26 red cards in the deck. $C(26, 5)$.

52. We want to figure out what m gives $C(m, 5) > 700$. You know the number on top must be larger than 5, since $C(5, 5) = 1$. Also, you know that

$C(m, 5) = \frac{P(m,5)}{5!}$

so multiply 700 by 5! and get 84000 now all you have to do is figure out what permutation it's closest to.

Notice $P(m, 5)$ is a product of 5 numbers less than or equal to m, so start by checking numbers greater than or equal to $84000^{1/5} < 10$. You'll discover that $P(10, 5)$ and$P(11, 5)$ aren't quite large enough, but $P(12, 5) = 95040 > 84000$ so the restaurant must have at least 12 appetizers.

64. 3 tasks: assign infield, then assign pitcher, then assign outfield/2nd base.

Infield: 4 positions, 4 players. $P(4, 4) = 4!$

Pitcher: 5 choices, only need one. $P(5, 1) = 5$

Outfield/2nd : 4 positions, 4 players. $P(4, 4) = 4!$

Final answer $4! \cdot 5 \cdot 4!$

65. Start by setting up 9 spaces to fill in the batting order, and put the pitcher in last.

1	2	3	4	5	6	7	8	9
								P

Now, suppose the first baseman bats 3rd. There's 7 slots left, and 7 people to fill them. Since order matters, there are 7! ways to make such a batting order.

1	2	3	4	5	6	7	8	9
		FB						P

Now let the first baseman bat 4th. Once again, you have 7! ways to make this sort of batting order.

1	2	3	4	5	6	7	8	9
			FB					P

Now add up the possible orders with the FB on 3rd, and those where he's 4th.

$7! + 7! = 2 \cdot 7!$ possible batting orders.

69. To get a hand of only 2 suits, we start by deciding which two we pick from: $C(4,2)$. Suppose we chose hearts and spades. Any hand with at least one card from each suit is viable, but a hand exclusively of one suit is not. We have a total of $C(26,5)$ possible hands. we subtract from this the ones only made up of one suit: $C(13,5)$ hands of just hearts, and $C(13,5)$ hands of just spades.

Our final answer becomes:

(pick 2 suits)(pick any 5 cards from the 26 minus the ways they only contain one suit)

$= C(4,2) \cdot \big(C(26,5) - C(13,5) - C(13,5)\big)$

72. Each pair of teams will play three games. There are $C(6,2)$ ways to pair up teams. So our final answer is

$C(6,2) \cdot 3$.

SECTION 5.6

1. a. Each flip has 2 possible outcomes. There are 6 flips. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6$.

1.b. Exactly 3 heads: whichever flips aren't heads, must be tails. So it's enough to pick which of the 3 flips are heads.

$C(6,3)$.

1.c. To have more heads than tails means, 4 H 2T, 5H 1T, or 6H 0T. So add the ways to get each of these results.

$C(6,4) + C(6,5) + C(6,6)$

1.d. At least 2 heads means 2, 3, 4,5, or 6 heads. That's a lot of calculations! I'd rather figure out how many ways don't do this, and figure out the difference between that and the total. The ways not to get at least 2 means the ways to get 1 or 0 heads. $= C(6,1) + C(6,0) = 7$

So take total outcomes 2^6 minus the 7 that don't work.

$2^6 - 7 = 57$

2. There are 2 moves down and 5 moves over. Determine which 2 are down out of the 7 total and we're done. $C(7,2)$

3. This problem is similar, but in this case we have a total of 9 moves to make, 4 of them down $C(9,4)$ possible paths.

5. a. A sample is a choice of any 3 apples from the 10 total. $C(10,3)$

5. b. Samples with all good apples are those where we pick 3 apples from just the 8 good ones. $C(8,3)$.

5. c. Samples with at least one rotten apple can be calculated using the complement of "at least one" = no rotten apples. No rotten apples is the same as all good apples. Since (all good) $\cup$ (at least one rotten) = every possible sample, $C(10,3) - C(8,3)$ gives the number with at least one rotten apple in it.

9. Break into 2 tasks, and applied generalized multiplication principle.

Task 1: paths A to C : $C(5,2)$ ways to get there.

Task 2: paths C to B: $C(4,2)$ ways to do this

Final answer= $C(5,2) \cdot C(4,2)$

10. a. This is a lot like 5. Total different samples is any choice of 5 from 100. $C(100,5)$

10. b. 2 defective fuses in a sample means the other 3 are good. So multiply $C(10,2)$ ways to get 2 bad fuses by $C(90,3)$ ways to get 3 good ones.

$C(10,2) \cdot C(90,3)$

10. c. "At least one bad" is the complement of "none bad."

Calculate the ways to get no bad fuses: $C(90,5)$ and subtract from total possible samples.

$C(100,5) - C(90,5) =$ total samples with at least one bad fuse.

11. To get a committee of 5 senators, no two from the same state, there are two ways to think of it:

1. Select the 5 states, then select a senator from each state.

$C(50,5)$ choices of 5 states$\cdot$2 choices from the first state$\cdot$2 choices from the second state$\cdot$2 choices from the 3rd 2 choices from the 4th state $\cdot$ 2 choices from the 5th state

Final answer $= C(50,5) \cdot 2^5$

2. Try to do this senator by senator, with order, then divide out by ordering:

$100 \cdot 98 \cdot 96 \cdot 94 \cdot 92 =$ ways to select 5 senators, no two from same state, in a given order $5! =$ ways these could be ordered.

Final answer= $\frac{100 \cdot 98 \cdot 96 \cdot 94 \cdot 92}{5!}$ which is the same as what you get the first way.

12. Getting 3 or more answers out of 5 right means:

Getting exactly 3 right $= C(5,3)$ OR getting exactly 4 right $= C(5,4)$ OR getting all 5 right $= C(5,5)$.

Add up these numbers to get all possible ways, since no two of these events could happen at the same time(you either got 3, 4, or 5 right)

$= C(5,3) + C(5,4) + C(5,5)$

15. We first select our jurors, then from whoever is left, select our alternates.

$C(20,12)$ choices of jurors. 12 people have been removed from the pool so only 8 are left to pick the alternates. $C(8,2)$ choices of alternates.

Final answer $C(20,12) \cdot C(8,2)$.

27. Several tasks here. do each separately then multiply together.

1. Pick 3 freshmen to stand in an ordered group $P(4,3)$

2.Pick 3 sophomores $P(5,3)$

3.Pick 3 juniors $P(6,3)$

4.Pick 3 seniors $P(7,3)$

Now order the classes. 4 groups of people ordered in a row. 4! ways to do this.

Final answer: $P(4,3) \cdot P(5,3) \cdot P(6,3) \cdot P(7,3)$

33. 13 ways to select the first denomination, those I take 3 of. I pick 4 of the 3 cards in that denomination $C(4,3)$.

12 ways to pick the 2nd denomination, and take 2 of the 4. $C(4,2)$.

Final answer: $13 \cdot C(4,3) \cdot 12 \cdot C(4,2)$

34. This problem is slightly different than above. Now I want to select 2 different denominations from the 13, and I'll pick 2 cards of each of these denominations.

$C(13,2) \cdot C(4,2) \cdot C(4,2)$

Then multiply by the way to pick the last card from a third denomination. $11 \cdot C(4,1)$.

Final: $C(13,2) \cdot C(4,2) \cdot C(4,2) \cdot 11 \cdot C(4,1)$

Don't worry about 40 and 41. They are ambiguous.