Answer Key 1

MATH 20–580: Linear Alg. and Diff. Eq. Name:____________________________

Final *December 16, 2005* Instructor:____________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 19 multiple choice questions worth 7 points each. You start with 17 points.

You may use a calculator if you wish.

HONOR CODE PLEDGE: As a member of the Notre Dame community, I will not participate in or tolerate dishonesty.

PLEASE SIGN:__________________________

1. a b c d •
2. a b • d e
3. • b c d e
4. a • c d e
5. a b • d e
6. a • c d e
7. • b c d e
8. a • c d e
9. a b c d •
10. a • c d e
11. a b • d e
12. a b c • e
13. a b c • e
14. a b • d e
15. a • c d e
16. a b c • e
17. a • c d e
18. a • c d e
19. a • c d e

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1.	a	b	c	d	e	11.	a	b	c	d	e
2.	a	b	c	d	e	12.	a	b	c	d	e
3.	a	b	c	d	e	13.	a	b	c	d	e
4.	a	b	c	d	e	14.	a	b	c	d	e
5.	a	b	c	d	e	15.	a	b	c	d	e
6.	a	b	c	d	e	16.	a	b	c	d	e
7.	a	b	c	d	e	17.	a	b	c	d	e
8.	a	b	c	d	e	18.	a	b	c	d	e
9.	a	b	c	d	e	19.	a	b	c	d	e
10.	a	b	c	d	e						

1. Suppose $y(x)$ is a solution to the initial value problem

$$(y - 3x^2)dx + (x - 1)dy = 0. \quad y(0) = 1.$$

Find $y(2)$.

(a) 11 (b) 3 (c) 5 (d) 9 (e) 7

2. Suppose that a spring is stretched 2 meters by a force of 100 newtons. The spring is hung vertically and a body with mass m=$\frac{1}{2}$ kg is attached to the end of the spring. Let u denote the displacement of the spring from the equlibrium position (measured downward). Suppose the system is set in motion with initial displacement $u_0 = .5$ meters, and initial velocity $u_0' = -10$ meters/sec. Assume the motion is undamped.

Find $u(t)$ when $t = \frac{\pi}{10}$ sec.

(a) $\frac{3}{2}$ meters (b) $\frac{1}{5}$ meters (c) $-\frac{1}{2}$ meters (d) $-\frac{1}{3}$ meters (e) $\frac{3}{5}$ meters

3. Suppose $y(x)$ is a solution to the initial value problem

$$y'' + y = \tan x, \quad y(0) = 0, \quad y'(0) = -1.$$

Find $y(\frac{\pi}{4})$.

Hints: $\int \sec x \; dx = \ln|\sec x + \tan x| + c$.

The general form of a solution is $c_1y_1(t) + c_2y_2(t) + y_p(t)$, and in this case $c_1 = c_2 = 0$.

$-\sin^2(x) = \cos^2(x) - 1$

(a) $-\frac{\ln(\sqrt{2}+1)}{\sqrt{2}}$ (b) $-\frac{\ln(\sqrt{2}+1)}{\sqrt{3}}$ (c) $-\frac{\ln(\sqrt{7}+1)}{\sqrt{3}}$ (d) $\frac{\ln(\sqrt{5}+1)}{\sqrt{2}}$ (e) $\frac{\ln(\sqrt{3}+2)}{\sqrt{2}}$

4. Find P and D such that $A = PDP^{-1}$ where $A = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) $P = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $P = \begin{bmatrix} -3 & 0 & 2 \\ 0 & -3 & 2 \\ 0 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(c) $P = \begin{bmatrix} 3 & 0 & 2 \\ 0 & -3 & 2 \\ 0 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) P and D do not exist.

(e) $P = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -2 \\ 0 & 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

5. For which values of k and l is the following matrix diagonalized?

$$\begin{bmatrix} 2 & k & 2 & -4 \\ 0 & 3 & 2 & -4 \\ 0 & 0 & 2 & l \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

(a) $k = 2 \ l = -2$.

(b) $k = 3, l = 2$

(c) $k = 1, l = 2$

(d) $k = 2, l = 3$

(e) Any values for k works and $l = 2$.

6. Which of the following sets of vectors is linear independent?

(a) $\{0, t, 1\}$ (in $\mathbb{P}_3$)

(b) $\{\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}\}$ (in the space of 2 by 2 matrices)

(c) $\{\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ -9 \end{bmatrix}\}$

(d) $\{\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} -3 & -2 \\ -1 & 4 \end{bmatrix}\}$ (in the space of 2 by 2 matrices)

(e) $\{1, (t+1)^2, t, t+1, t^2\}$ (in $\mathbb{P}_3$)

7. Suppose $y(x)$ is a solution to the initial value problem

$$3y'' + y' - 2y = 2\cos x, \quad y(\frac{\pi}{2}) = \frac{1}{13}, \quad y'(\frac{\pi}{2}) = \frac{5}{13}.$$

Find $y(0)$.
Hint: The general form of a solution is $c_1 y_1(t) + c_2 y_2(t) + y_p(t)$, and in this case $c_1 = c_2 = 0$.

(a) $-\frac{5}{13}$ (b) $\frac{4}{11}$ (c) $\frac{6}{19}$ (d) $-\frac{3}{17}$ (e) $\frac{3}{16}$

8. Find the determinant of

$$A = \begin{bmatrix} 2 & 0 & 0 & 12 & 2 \\ 0 & 0 & 2 & -2 & 0 \\ 3 & -2 & 1 & 4 & -5 \\ 0 & 0 & 0 & 2 & 0 \\ -2 & 0 & 0 & 5 & 4 \end{bmatrix}.$$

(a) Does not exist.

(b) 96

(c) 16

(d) 128

(e) -32

9. Find all solutions to $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 0 & 3 & 3 & 0 \\ 3 & 1 & 7 & 3 & 8 \\ -1 & 3 & -9 & 0 & -18 \\ 1 & 1 & 1 & -1 & 4 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 11 \\ 0 \end{bmatrix}$$

and $[A|\mathbf{b}]$ row reduces to $\begin{bmatrix} 1 & 0 & 3 & 0 & 6 & -2 \\ 0 & 1 & -2 & 0 & -4 & 3 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) $c_1 \begin{bmatrix} -6 \\ 4 \\ 0 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, where c_1 is any real.

(b) There are no solutions.

(c) $c_1 \begin{bmatrix} 6 \\ -4 \\ 0 \\ -2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ 0 \\ -1 \\ 0 \end{bmatrix}$, where c_1, c_2 are any reals.

(d) $\begin{bmatrix} -2 \\ 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

(e) $c_1 \begin{bmatrix} -6 \\ 4 \\ 0 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -3 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, where c_1, c_2 are any reals.

10. Let A be a 4×4 matrix of rank 3. Which of the following is *true*?

(a) The dimension of the null space is 3.

(b) When $A\mathbf{x} = \mathbf{b}$ is consistent $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

(c) The first 3 rows of A form a basis for the row space of A.

(d) $A\mathbf{x} = \mathbf{b}$ is never consistent.

(e) $A\mathbf{x} = \mathbf{b}$ is always consistent.

11. You will need \$50,000 each year to live on after you retire, and you plan on living 30 years after retirement. Your retirement account will earn 5% interest per year. Assume that you spend money continuously and that interest is paid continuously. How much money must be in the retirement account when you retire?

(a) \$299,367 (b) \$345,874 (c) \$776,870 (d) \$544,845 (e) \$942,867

12. Suppose $y(x)$ is a solution to the initial value problem

$$y'' + 4y' + 4y = 0, \quad y(2) = \frac{1}{e^4}, \quad y'(2) = \frac{2}{e^4}.$$

Find $y(\frac{1}{2})$.

(a) $-\frac{7}{e}$ (b) $\frac{3}{e}$ (c) $\frac{2}{e}$ (d) $-\frac{5}{e}$ (e) $\frac{3}{e}$

13. Let $\mathbf{y} = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 13 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ -1 \\ 2 \end{bmatrix}$, and $\mathbf{v}_2 = \begin{bmatrix} -4 \\ 1 \\ 0 \\ 3 \end{bmatrix}$. Find the distance from $\mathbf{y}$ to the subspace of $\mathbf{R}^4$ spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$.

(a) 9 (b) 7 (c) 5 (d) 8 (e) 10

14. Consider the transformation T from $\mathbb{P}_2$ to $\mathbb{P}_3$ given by $T(f(t)) = (2t+1)f(t)$. Let $\mathcal{E}_i = \{1, t, t^2, t^3, \ldots, t^i\}$ be the standard basis for $\mathbb{P}_i$ Find the matrix for T relative to the $\mathcal{E}_2$ and $\mathcal{E}_3$.

(a) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

(d) T is not a linear transformation and hence such a matrix does not exist.

(e) $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

15. Let A be a 4×6 matrix whose column space is all of $\mathbb{R}^4$. Which of the following is *false*?

(a) The rank of A is 4.

(b) The row space of A has dimension 2.

(c) When $A\mathbf{x} = \mathbf{b}$ is consistent there are infinitely many solutions.

(d) The null space of A has dimension 2.

(e) $A\mathbf{x} = \mathbf{b}$ is always consistent.

16. Let $\mathbf{u} = \begin{bmatrix} k \\ k \\ 1 \end{bmatrix}$, and $\mathbf{v} = \begin{bmatrix} k \\ 5 \\ 6 \end{bmatrix}$. For what values of k are $\mathbf{u}$ and $\mathbf{v}$ orthogonal?

(a) 1.7 and -2.3 (b) -3 and -5 (c) -1.5 and 3.7 (d) -2 and -3 (e) 2 and -5

17. Suppose that a spring is stretched 2 meters by a force of 100 newtons. The spring is hung vertically and a body with mass m=$\frac{1}{2}$ kg is attached to the end of the spring. Let u denote the displacement of the spring from the equlibrium position (measured downward). Suppose the system is set in motion with initial displacement $u_0 = .5$ meters, and initial velocity $u_0' = -10$ meters/sec. Assume the motion is damped with damping coefficient 6 newtons.

Find $u(t)$ when $t = \frac{\pi}{16}$ sec.

(a) $\frac{3}{8}e^{-\frac{3\pi}{8}}$ meters (b) $-\frac{7}{8}e^{-\frac{3\pi}{8}}$ meters (c) $-\frac{7}{8}e^{-\frac{5\pi}{8}}$ meters

(d) $-\frac{5}{8}e^{-\frac{7\pi}{8}}$ meters (e) $\frac{5}{8}e^{-\frac{5\pi}{8}}$ meters

18. Find the best solution to $\begin{bmatrix} 1 & 0 \\ 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$.

(a) $\begin{bmatrix} 2 \\ 2.5 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$

(e) There is no best solution.

19. Let $A = \begin{bmatrix} 1 & 0 & 3 & 3 & -2 \\ 2 & 1 & 5 & 3 & 0 \\ -1 & 3 & -6 & 0 & 2 \\ 1 & 1 & 2 & -1 & 3 \end{bmatrix}$. Which of the following is the reduced row echelon form of A?

(a) $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 3 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$