Answer Key 1

MATH 20–580: Linear Alg. and Diff. Eq. Name:_____

Exam I *September 22, 2005* Instructor:_____

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 6 multiple choice questions worth 8 points each and 4 partial credits problems worth 12 points each. You start with 4 points. On the partial credit problems try to simplify your answer and indicate your final answer clearly. *You must show your work and all important steps to receive credit.*

You may use a calculator if you wish.

Honor Code Pledge: As a member of the Notre Dame community, I will not participate in or tolerate dishonesty.

Please Sign:_____

1. • b c d e
2. • b c d e
3. • b c d e
4. • b c d e
5. • b c d e
6. • b c d e

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1. [a] [b] [c] [d] [e]
2. [a] [b] [c] [d] [e]
3. [a] [b] [c] [d] [e]
4. [a] [b] [c] [d] [e]
5. [a] [b] [c] [d] [e]
6. [a] [b] [c] [d] [e]

1. Let A be a matrix which is row reduced to $\begin{bmatrix} 1 & -3 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. Which of the following statements is *false*.

   (a) Every column of A can be written as a linear combination of the others.

   (b) The transformation $T(\mathbf{x}) = A\mathbf{x}$ is onto.

   (c) The columns of A are linearly dependent.

   (d) $\mathbf{a}_4 = \mathbf{a}_1 - 2\mathbf{a}_3$, where $\mathbf{a}_i$ is the ith column of A.

   (e) For every $\mathbf{b}$, the equation $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

2. The 7×5 matrix A row reduces to a matrix with a pivot in every column. Which of the following is *true*

   (a) The transformation $T(\mathbf{x}) = A\mathbf{x}$ is one-to-one.

   (b) A is invertible.

   (c) The columns of A span $\mathbb{R}^7$

   (d) For every $\mathbf{b}$, the equation $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

   (e) A is row reducible to I_5.

3. Let $A = \begin{bmatrix} 1 & 37 & -1 & 12 \\ 100 & 5 & -4 & 7 \\ -27 & 9 & 0 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 11 & 3 & -1 \\ 0 & 5 & -4 \\ -27 & 9 & 1 \end{bmatrix}$. Then which of the following is *true*?

(a) The entry in the 2th row, 3th column of BA, is -20.

(b) AB exists.

(c) BA exists and is 4×3.

(d) The entry in the 2th row, 3th column of BA, does not exist.

(e) The entry in the 2th row, 3th column of BA, is 12.

4. Let $\mathbf{v_1} = \begin{bmatrix} -6 \\ 7 \\ 2 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$.

Consider the following statements.
(I:) All 3 vectors lie on a straight line thru the origin.
(II:) All 3 vectors lie on a plane (but not a straight line) thru the origin.
(III:) The 3 vectors do not lie on a plane thru the origin.
(IV:) The set of vectors $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is linearly independent.
(V:) $det(A) = 0$ where

$$A = \begin{bmatrix} -6 & 3 & 4 \\ 7 & 2 & -1 \\ 2 & 4 & 2 \end{bmatrix}.$$

(a) Statements II and V are correct.

(b) Statements I and V are correct.

(c) Statement III is correct, but statement V is false.

(d) Statement III is false and IV is correct.

(e) Statements II and IV are correct.

5. Let A be the standard matrix for a linear transformation $T : \mathbf{R}^4 \to \mathbf{R}^2$. Suppose $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is a linearly independent set of vectors in $null(A)$. We can conclude

   (a) The linear transformation T is not onto.

   (b) The equation $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is inconsistent.

   (c) The dimension of $null(A)$ is 3.

   (d) The number of basic variables for the equation $A\mathbf{x} = \mathbf{0}$ is at least 2.

   (e) The linear transformation T is 1-1.

6. Suppose A is the standard matrix for an onto linear transformation $T : \mathbf{R}^n \to \mathbf{R}^n$. What can one conclude about $det(A)$?

   (a) $det(A)$ is not zero

   (b) $det(A)$ is either 1 or -1

   (c) $det(A)$ might be any real number

   (d) $det(A) > 0$

   (e) $det(A) < 0$

7. Find the general solution to the following system of linear equations (be sure to show your work):

$$\begin{array}{rcrcrcrcl} x_1 & - & 2x_2 & + & x_3 & + & 3x_4 & = & 3 \\ 2x_1 & - & 4x_2 & & & + & 6x_4 & = & 4 \\ 2x_1 & - & 4x_2 & + & 2x_3 & + & 4x_4 & = & 6 \\ & & & & x_3 & - & x_4 & = & 1 \end{array}$$

8. Find a matrix B such that BA is in reduced echelon form where

$$A = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 0 & -1 \\ -\frac{3}{2} & \frac{1}{4} & 0 & \frac{1}{2} \end{bmatrix}$$

(be sure to show all your work). Is B the inverse of A? Why or why not?

9. Suppose

$$A = \begin{bmatrix} 2 & 2 & 3 & 2 \\ 1 & 2 & 1 & 4 \\ 1 & 0 & 2 & -2 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

and the reduced echelon form for A is

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Find a basis for $null(A)$.
(b) Find a basis for $col(A)$.

10. Suppose

$$A = \begin{bmatrix} 5 & 5 & 2 \\ 2 & 4 & 6 \\ 2 & 3 & 1 \end{bmatrix}.$$

Compute $det(A)$ by finding the echelon form of A. (Show your work for each step.)