# Recovery of Quantized Ballistic Conductance in a Periodically Modulated Channel

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For a periodically modulated electron channel, conductance quantization characteristic of a simple quantum point contact is recovered. The value of the quantized conductance is, however, no longer a monotonically increasing function of energy. Comparison with the band structure of the corresponding infinite channel shows a direct correspondence between the index of the conductance plateau and the number of positive-velocity bands at a given energy. The results persist in the presence of an applied magnetic field and lead to a prediction of nonmonotonic steps in the integer quantum Hall resistance for such structures.

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Quantization of the conductance of a quantum point contact (QPC), such as those fabricated by lateral confinement of a two-dimensional electron gas in a semiconductor, is now well known and well understood [1-3]. The quantization is due to the creation of lateral subband modes, analogous to waveguide modes, and the fact that each mode carries the same amount of current. The linear-response conductance for a QPC is simply $G(E) = N(E)2e^2/h$, where $N(E)$ is an integer function of the energy which counts the number of traveling modes with energy below $E$. The conductance increases monotonically in steps as additional modes become available. If the ballistic channel is patterned with further features, bends, constrictions, or other obstructions, conductance quantization is lost and a complicated structure for $G(E)$ emerges due to the details of quantum interference and backscattering in the channel [4,5].

In this Letter we show that if a ballistic channel has a periodically modulated structure, the quantization in $G(E)$ is recovered but it is no longer a monotonic function of energy. We numerically calculate transmission coefficients and the linear-response conductance of such a channel. We also calculate the energy band structure for the corresponding infinite modulated channel and show that simple features of the band structure explain the conductance behavior [6,7].

We examine the structure shown schematically in Fig. 1. A channel of width $d$ is periodically narrowed to a width of $d-h$. The period of the modulation is $a$, the number of narrow regions is $N$, and the length of each narrow region is $w$. We present results for the particular case where $d/a = 2.0$, $h/a = 0.6$, and $w/a = 0.4$. A magnetic field of magnitude $B$ is applied perpendicular to the plane in the $\hat{\mathbf{z}}$ direction. The Landau gauge is chosen for the vector potential so $\mathbf{A} = -By\hat{\mathbf{x}}$. We adopt a single-band effective-mass model with an effective mass appropriate for GaAs, $m^*/m_0 = 0.067$, and ignore spin throughout. Hard wall potentials are assumed to define the channel modulation while the potential inside the channel is taken to be zero [8].

We solve the two-dimensional time-independent Schrödinger equation,

$$\left[\frac{-\hbar^2}{2m^*}\nabla^2 + \frac{ie\hbar By}{m^*}\frac{\partial}{\partial x} + \left(\frac{e^2B^2y^2}{2m^*}\right) + V_0(x,y)\right]\psi(x,y) = E\psi(x,y)\,. \tag{1}$$

To calculate the conductance of a channel of finite length and $N$ periods, we solve Eq. (1) to obtain the complex energy-dependent transmission and reflection amplitudes for each transverse mode (defined in the wide regions). We solve directly for the wave function and transmission amplitudes in one unit cell (marked by dashed lines in Fig. 1), then use a scattering matrix cascading method [9] to obtain transmission and reflection amplitudes for the whole structure. Transmission and reflection into evanescent modes must be included in the cascading process. The conductance in the linear response regime is then obtained using the Landauer equation [10],

$$G = \frac{2e^2}{h}\mathrm{Tr}(tt^\dagger)\,, \tag{2}$$

where $t_{i,j}$ is the transmission amplitude from mode $j$ into mode $i$ for the whole structure.

The numerical solution of Eq. (1) for the unit cell with open boundaries is accomplished using the quantum transmitting boundary method [11], a numerical algorithm we have developed based on the finite element method for solving the two-dimensional Schrödinger equation for current-carrying states. We employ a recent extension of the method to include the case of an applied magnetic field [12].

We compare the conductance for the finite system with $N$ periods of modulation with the band structure of the infinite periodic system. For the infinite system, we can use the Bloch theorem and look for a solution of the form $\psi_{n,k}(x,y) = e^{ikx}u_{n,k}(x,y)$ where $u_{n,k}(x,y)$ is the periodic part of the Bloch function. We solve Eq. (1) with this

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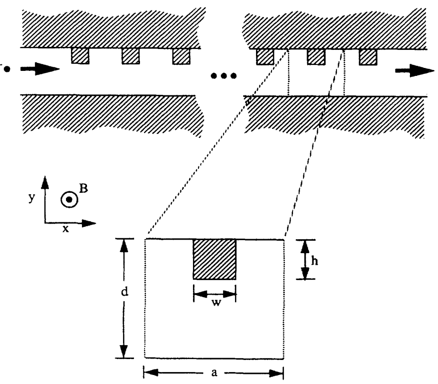


FIG. 1. The geometry of the periodically modulated quantum channel.

substitution as an eigenvalue problem for $E_n(k)$ and $u_{n,k}(x,y)$. We use the finite element method to achieve numerical discretization over the unit cell. Meshes of up to 5151 modes were used to achieve convergence at high values of magnetic field.

Figure 2 shows the calculated conductance and transmission coefficients $T_j = \sum_i |t_{i,j}|^2$ for a finite channel and the band structure for an infinite channel when no magnetic field is applied. Energy is expressed in units of $E_1$, the energy of the first transverse mode (subband) in the wide regions. Figure 2(a) shows the transmission of a channel with three unit cells (three constrictions). The total transmission for various incoming modes are shown separately. The energy band structure for the infinite modulated channel is shown for the first Brillouin zone in Fig. 2(b). Energy is plotted on the horizontal axis so that the relationship to the conductance results below is clear. Figure 2(c) shows the conductance calculated from Eq. (2) for a long channel with $N=85$ periods. Figure 2(d) shows the transmission for individual modes in the long modulated structure [13].

For only three constrictions in the channel the total transmission, shown as the solid curve of Fig. 2(a), which is proportional to the conductance, is not quantized. This is to be expected because of mode mixing due to the abrupt narrowing of the channel [2,3]. The conductance of the long modulated channel, shown in Fig. 2(c), is striking in that the conductance is essentially quantized in units of $2e^2/h$ [14]. Unlike the usual quantization of QPC ballistic conductance, however, the conductance does not increase monotonically but rather steps up *and down* between quantized levels, sometimes going to zero [15]. The ballistic conductance of the very long channel can be written as $G(E) = N_c(E) 2e^2/h$, where $N_c(E)$ is the integer index corresponding to the quantized conductance plateau for energy $E$.

The conductance quantization in Fig. 2(c) can be understood by examining the band structure shown in Fig. 2(b). For each value of the energy, define an integer $N_b^+(E)$ to be the number of energy bands [distinct $E_n(k)$

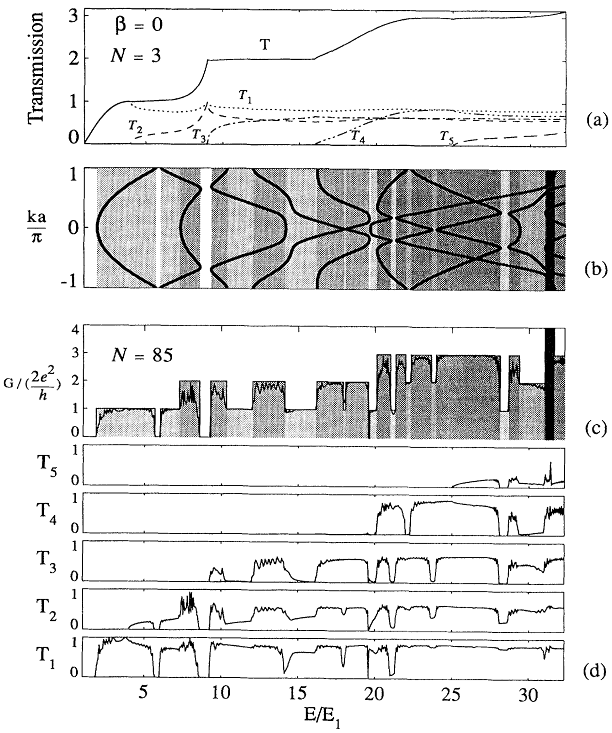


FIG. 2. The modulated channel in no applied magnetic field. (a) Transmission coefficients for a short channel with three unit cells ($N=3$). (b) The energy band structure for the infinite periodic channel. The grey scale shadings indicate the number of individual energy bands with positive group velocity in each energy region (white=0, lighter=1, light=2, dark=3, darker=4). (c) Conductance for a long channel with 85 unit cells ($N=85$). Different grey scale shadings indicate the index of the quantization plateau (white=0, lighter=1, light=2, dark=3, darker=4). The correspondence of the grey scale schemes in (b) and (c) illustrates that the conductance of the finite channel is related to the number of positive-velocity bands in the band structure of the infinite channel. (d) Transmission coefficients of individual modes for the long modulated channel.

curves] with positive group velocity (slope). $N_b^+(E)$ is also a nonmonotonic function; it is zero in energy gaps and steps up and down as a function of energy. By comparing Figs. 2(b) and 2(c) we see that, in fact, $N_c(E) = N_b^+(E)$. The number of positive velocity bands for the infinite system yields the quantization of the conductance in the periodically modulated finite system. The shaded regions of the figures illustrate this correspondence. Each value of $N_c(E) = N_b^+(E)$ is represented by a different grey scale value.

For a straight channel or a short ballistic point contact, $N_b^+(E)$ is simply the subband number and increases monotonically. The well known cancellation of velocity and density-of-states factors leads to identical current being carried in quasi-one-dimensional subband. Thus $N_c(E)$ is quantized and monotonic in the same manner.

The nonmonotonic behavior of $N_b^+(E)$ in the periodically modulated channel is due to the significant amount of reflection and mode mixing caused by even a single constriction in the channel. Transport is by no means adiabatic. This mode mixing in the finite channel results in band mixing in the band structure of the infinite channel. The band mixing results in the appearance of forbidden gaps and allowed energy regions with differing numbers of energy bands. Just as subbands in a straight channel each carry the same amount of current, Bloch bands in the periodic structure each carry the same current.

These results have no strictly one-dimensional analog. The band structure in a one-dimensional periodic system has the same number of bands (two, one with positive velocity and one with negative velocity) in each allowed region. $N_b^+(E)$ in one dimension only takes the values of 1 in a band or 0 in a gap. The integer values $(0,1,2,3,\ldots)$ of $N_b^+(E)$ for the modulated channel manifest the two-dimensional character of the channel mixing with the quasi-one-dimensional character of the current flow.

It is important to note that the transmission of individual modes is *not quantized* but the total transmission is. Figure 2(d) shows that for no individual mode is the conductance quantized. The quantization occurs as the various modes are mixed by the periodic scattering.

The recovery of ballistic conductance quantization persists in the presence of a perpendicular magnetic field. Figure 3 shows the conductance and band structure calculated for the case of a moderately high magnetic field, $\beta \equiv (eB/\hbar)ad = 24.3$ (for a unit cell with $a = 200$ Å, this corresponds to $B = 20$ T) [16]. Here the conduction can be described in terms of edge states and the energy is naturally expressed in units of the first bulk Landau level, $E_L = \hbar(eB/2m^*)$.

In the limit of extremely high magnetic fields, all transport would be through edge states and the suppression of backscattering between edge states on opposite sides of the channel would guarantee monotonically increasing and quantized conductance [17]. The interesting result here is that for intermediate field strengths (or for higher edge-state indices), where backscattering can still occur between edge states, the conductance is nevertheless quantized. Again, however, the quantization is nonmonotonic and related directly to the number of positive-velocity bands in the band structure for the corresponding infinite system. A step *down* in conductance as energy is increased means that an edge state which was contributing to the conduction has now been turned off at the higher energy because it resonantly backscatters from the constrictions.

Notice in Fig. 3(c) that the conductance is lowered by one unit for energies just above $6E_L$. Examination of the individual mode transmission coefficients in Fig. 3(d) shows that it is the *first* edge state (with a slight admixture of the third) which has been resonantly reflected. For this energy range the second and third edge states are

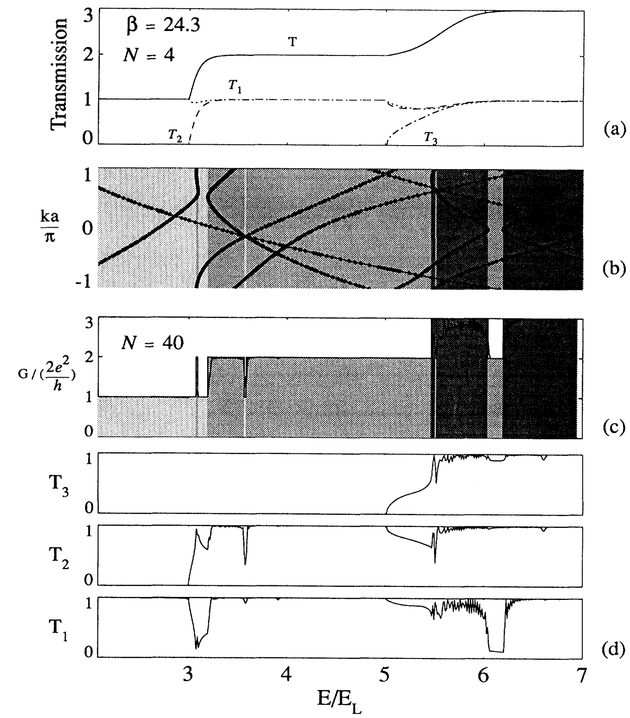


FIG. 3. The modulated channel in an applied magnetic field. (a) Transmission coefficients for a short channel with four unit cells ($N=4$). (b) The energy band structure for the infinite periodic channel. The grey scale shadings indicate the number of individual energy bands with positive group velocity in each energy region (white=0, lighter=1, light=2, dark=3, darker=4). (c) Conductance for a long channel with 40 unit cells ($N=40$). Different grey scale shadings indicate the index of the quantization plateau (white=0, lighter=1, light=2, dark=3, darker=4). The correspondence of the grey scale schemes in (b) and (c) illustrates that the conductance of the finite channel is related to the number of positive-velocity bands in the band structure of the infinite channel. (d) Transmission coefficients of individual modes (edge states) for the long modulated channel.

almost entirely transmitted but the first and outermost edge state is reflected. This selective reflection of edge states is similar to the experimental results of Müller *et al.* [18], who used an applied potential from a metal gate to reflect individual edge states. The consequence of this reflection was a deviation from the usual integer quantum Hall effect (IQHE) plateaus, a deviation understandable in the edge-state picture of the IQHE [17]. The reflection of selective edge states seen in our calculation for a modulated channel suggests that similar IQHE deviations, steps up and down between Hall resistance plateaus, should be observable in these geometries.

In conclusion, we have studied the ballistic transport properties of a periodically modulated channel. Our results show that a long modulated channel has a ballistic conductance which is quantized, but a nonmonotonic function of energy. The index of a quantized conductance plateau has a one-to-one correspondence to the number of positive-velocity states in the energy band structure for the corresponding infinite modulated channel. This phenomenon persists at high magnetic field, where it can be interpreted as resonant reflection of particular edge states and should produce anomalous IQHE behavior.

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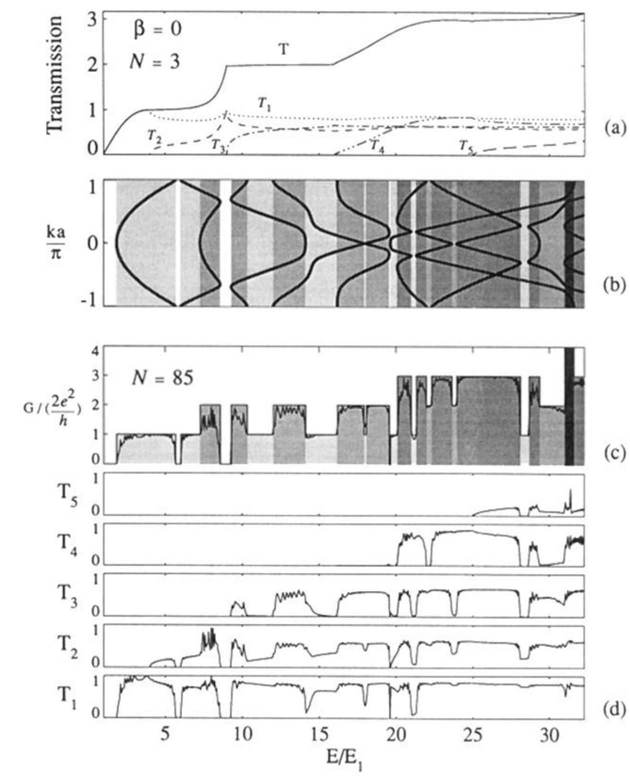


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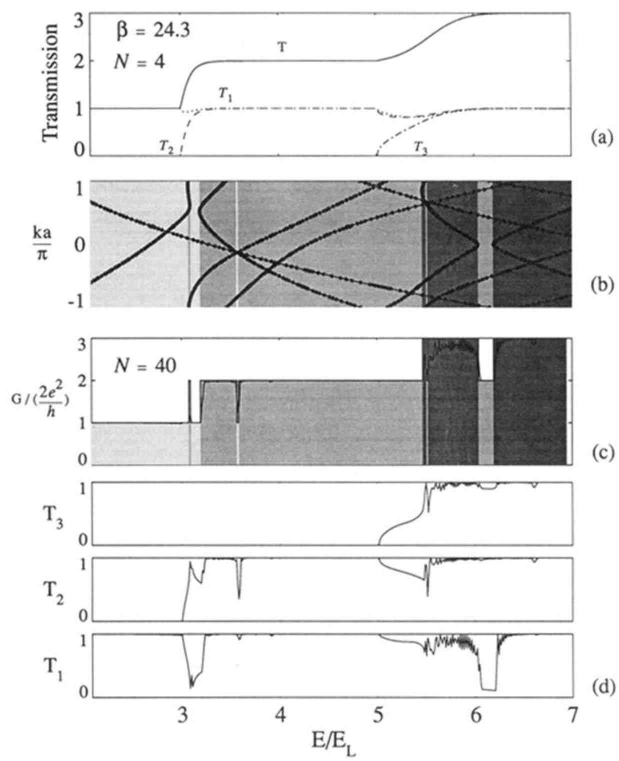


FIG. 3. The modulated channel in an applied magnetic field. (a) Transmission coefficients for a short channel with four unit cells ($N=4$). (b) The energy band structure for the infinite periodic channel. The grey scale shadings indicate the number of individual energy bands with positive group velocity in each energy region (white=0, lighter=1, light=2, dark=3, darker=4). (c) Conductance for a long channel with 40 unit cells ($N=40$). Different grey scale shadings indicate the index of the quantization plateau (white=0, lighter=1, light=2, dark=3, darker=4). The correspondence of the grey scale schemes in (b) and (c) illustrates that the conductance of the finite channel is related to the number of positive-velocity bands in the band structure of the infinite channel. (d) Transmission coefficients of individual modes (edge states) for the long modulated channel.