Math 60-650-01: Spring 2007

Basic Partial Differential Equations

Instructor: Yongtao Zhang (yzhang10@nd.edu)

Textbook

1. Partial Differential Equations, Lawrence C. Evans, American Mathematical Society, ISBN: 0-8218-0772-2, 1998.
2. Numerical Solution of Time-Dependent Advection-Diffusion-Reaction Equations, Willem Hundsdorfer, Jan Verwer, Springer-Verlag, ISBN: 3-540-03440-4, 2003.

Course Description

This is an introduction course for graduate students to learn fundamental concepts, theory and techniques in partial differential equations. The first part of this course will cover some basic theory for elliptic, parabolic and hyperbolic PDEs, and the second part will introduce some basic numerical techniques in applying PDEs to model real problems, such as reaction-diffusion equations arising from developmental biology.

Topics

1. **Four important Linear PDE**
   - Transport equation; Laplace's equation; Heat equation; Wave equation.
2. **Nonlinear First-Order PDE**
   - Characteristics of PDE; Hamilton-Jacobi equations; Conservation Laws.
3. **Ways to Represent Solutions**
   - Separation of variables; Similarity solutions; Transform methods.
   - Converting nonlinear into linear PDE.
4. **Reaction Diffusion, Chemotaxis, and Nonlocal Mechanisms**
   - Simple random walk and derivation of the diffusion equation.
   - Reaction-Diffusion equations.
   - Chemotaxis.
   - Nonlocal effects and long range diffusion.
5. **Basic Spatial Discretizations**
   - Advection-Diffusion discretizations by Finite Difference and Finite Element.
   - Moving Grids.
6. **Time Integration Methods for Reaction-Diffusion equations**
   - Method of Lines.
   - Runge-Kutta methods; Linear Multiple-step methods, Monotone ODE methods; Variable step size control.
   - Splitting methods (Operator Splitting, LOD, ADI, IMEX, ETD).

References

[1] J.D. Murray, Mathematical Biology (I, II), Springer, 2002.

[2] J. Ockendon, S. Howison, A. Lacey, and A. Movchan, Applied Partial Differential Euqations, Oxford, 2003.

[3] L. Edelstein-Keshet, Mathematical Models in Biology, SIAM, 2005.