Math 60790 (Numerical PDEs), Spring 2008

Homework 1 (Due Feb 18, 2008)

1. Let f be a real function with the Fourier series $f(x) = \frac{1}{\sqrt{2\pi}} \sum_{\omega=-\infty}^{\infty} \hat{f}(\omega) e^{i\omega x}$. Prove that $S_N = \frac{1}{\sqrt{2\pi}} \sum_{\omega=-N}^{N} \hat{f}(\omega) e^{i\omega x}$ is real for all N.

2. Consider the saw-tooth function $v(x) = \frac{1}{2}(\pi - x)$ for $0 < x \le 2\pi$, $v(x) = v(x+2\pi)$, and the partial Fourier sums $v_N(x) = \sum_{\omega=1}^{N} \frac{\sin \omega x}{\omega}$. Write a program to compute $v(x) - v_N(x)$ and plot $v_N(x)$ and $v(x) - v_N(x)$ on $0 \le x \le 2\pi$, for $N = 10, 100, 1000, 10000$. Analytically one can show that $v(x) - v_N(x) = R((N+1/2)x) + O(\frac{|x| + 1/N}{N})$, where $R(y) = \frac{\pi}{2} - \int_0^y \frac{\sin t}{t} dt$. Verify this theoretical error estimate numerically for some x values chosen by yourself.

3. Derive estimates for $\left| \left(D - \frac{\partial^3}{\partial x^3} \right) e^{i\omega x} \right|$ where $D = D_+^3,\ D_0 D_+ D_-$.

4. Compute $\| D_+ D_- \|_h$.