# Reflections on Skolem's Paradox

The Löwenheim-Skolem theorems say that if a first-order theory has infinite models, then it has models which are only countably infinite. Cantor's theorem says that some sets are uncountable. Together, these two theorems induce a puzzle known as *Skolem's Paradox:* the very axioms of (first-order) set theory which prove the existence of uncountable sets can themselves be satisfied by a merely countable model.

Ever since this paradox was formulated in the early 1920's, philosophers have argued that issues of deep philosophical importance hang on the paradox's resolution. Skolem himself used the paradox to argue that set theory provides an inadequate foundation for mathematics. Later authors have used the paradox to argue that "every set is countable from some perspective" (Wang) or that Quine's theory of ontological reduction is hopelessly flawed (Grandy and Chihara). Most recently, Hilary Putnam has claimed that the paradox has, in his words, "profound implications for the great metaphysical dispute about realism which has always been the central dispute in the philosophy of language."

The present dissertation examines Skolem's Paradox from three perspectives. After a brief introduction, chapters two and three examine a number of different formulations of the paradox in order to disentangle the roles which set theory, model theory, and philosophy play in these formulations. In these two chapters, I accomplish three things. First, I clear up some of the mathematical ambiguities which have all too often infected discussions of Skolem's Paradox. Second, I isolate a key philosophical assumption upon which Skolem's Paradox rests, and I show why this assumption has to be false. Finally, I argue that there is no *single* explanation as to how a countable model can satisfy the axioms of set theory (in particular, no explanation in terms of quantifier-ranges can ever be fully adequate).

In chapter four, I turn to a second puzzle. Why, even though philosophers have known since the 1920's that Skolem's Paradox has a relatively simple technical solution, have they continued to find this paradox so troubling? I argue that philosophers' attitudes towards Skolem's Paradox have been shaped by the acceptance of some fairly specific claims in the philosophy of language. I then tackle these philosophical claims head on. In some cases, I argue that the claims are ill-motivated, as they depend on an incoherent account of mathematical language. In other cases, I argue that the claims are so powerful that they render Skolem's Paradox trivial: once these claims are on the table, set-theoretic problems appear immediately and Skolem's Paradox itself is reduced to mere technical window-dressing. In any case, examination of the philosophical underpinnings of Skolem's Paradox renders that paradox decidedly unparadoxical.

Finally, in chapter five, I turn away from generic formulations of Skolem's Paradox to examine Hilary Putnam's "model-theoretic argument against realism." I show that Putnam's argument involves mistakes of both the mathematical and the philosophical variety and that these two types of mistakes are closely related. Along the way, I clear up some of the mutual charges of question begging which have characterized discussions between Putnam and his critics. In the end, I conclude that Putnam's model-theoretic argument is simply a failure.