Soc 63993, Homework #7 Answer Key:
Nonlinear effects/ Intro to path analysis

Richard Williams, University of Notre Dame, https://www3.nd.edu/~rwilliam/

Last revised February 20, 2015

Problem 1. The files *nonlinhw.do* and *nonlinhw.dta* will generate the computer runs you need for this problem. Copy them from the course web page. You will also need to install `curvefit`, available from SSC. (You will need to refer to curvefit's help file so you know what the functions are.) Run the program a few lines at a time; otherwise you will always be erasing your graphs.

There are 4 variables in nonlinhw.dta: X1 (the IV), Y1, Y2, and Y3 (the DVs). The Stata program does scatterplots of X1 versus each DV and then generates other graphs that model the nonlinear relationship. For each DV in turn, you are to do the following:

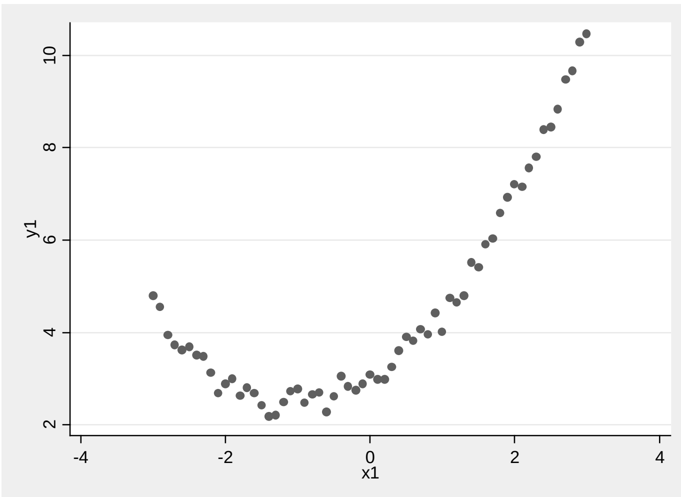
- Examine each scatterplot. Explain why the relationship is nonlinear and what type of nonlinearity appears to be present. Put another way, explain the rationale for the followup graph of the nonlinear relationship.
  - For Part I only, show a different set of Stata commands that could graph the nonlinear relationship.
  - For Parts I and II only, show how the same models could be estimated using the `regress` and/or `glm` commands.
  - For Y3 only, two different Curvefits are presented (Parts III and IV). Explain why, based on the graphics only, it would be difficult to decide which nonlinear specification was most appropriate, and how theory might help you to choose.
- Discuss what problems result from a linear (mis)specification. The graphs will help you here.
- For Parts I, II, III, present a substantive example, real or hypothetical, that the model you have estimated might be appropriate for. Explain why it is appropriate. Do not use any of the examples already given in class.

First off, here is nonlinhw.do:

```
version 12.1
use https://www3.nd.edu/~rwilliam/statafiles/nonlinhw.dta, clear

********** Part 1.
* Plot of x1 with y1
estimates clear
scatter y1 x1, scheme(sj)
curvefit y1 x1, f(1 4)
* HW: Show another way to graph this model
* HW: Show how to estimate this model using regress and/or glm

********** Part 2.
* Plot of x1 with y2
estimates clear
scatter y2 x1
curvefit y2 x1, f(1 0)
* HW: Show how to estimate this model using regress and/or glm

********** Part 3.
* Plot of x1 with y3.
estimates clear
scatter y3 x1
mkspline xle0 0 xgt0 = x1, marginal
reg y3 x1
predict linear
reg y3 xle0 xgt0
predict spline
scatter y3 x1 || line linear x1 || line spline x1, sort scheme(sj)

******** Part 4.
* As this shows, a polynomial model would also be plausible for y3.
* In practice, it is often hard to tell just from the scatterplot what
* transformation is best, so theory is important.
estimates clear
curvefit y3 x1, f(1 4)
```

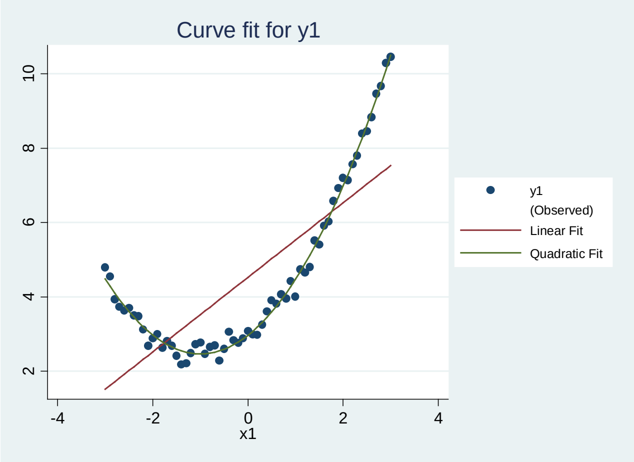
The scatterplot of X1 with Y1 is

```
. use https://www3.nd.edu/~rwilliam/statafiles/nonlinhw.dta, clear
. ********** Part 1.
. * Plot of x1 with y1
. estimates clear
. scatter y1 x1, scheme(sj)
```



The U-shaped, curvilinear form suggests that a polynomial model is called for. There appears to be one “bend” so the model should include terms for X1 and $X1^2$. The `curvefit` command estimates the linear and quadratic models and generates the graph.

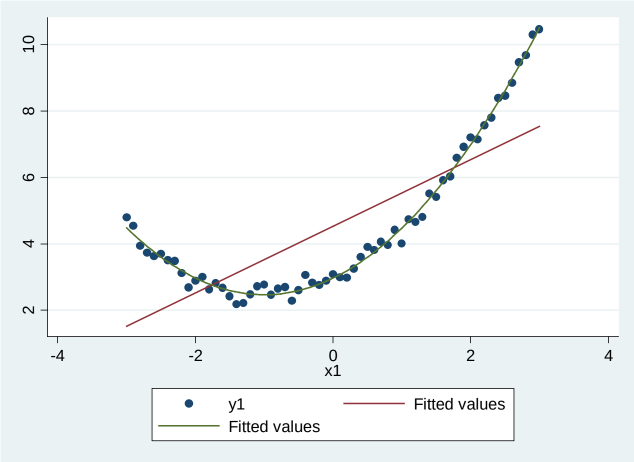
```
. curvefit y1 x1, f(1 4)

Curve Estimation between y1 and x1

------------------------------------------
    Variable |    Linear       Quadratic
-------------+----------------------------
b0           |
       _cons |   4.5251426      2.966442
             |       24.71         78.56
             |      0.0000        0.0000
-------------+----------------------------
b1           |
       _cons |     1.00287       1.00287
             |        9.64         70.16
             |      0.0000        0.0000
-------------+----------------------------
b2           |
       _cons |                .50280663
             |                    55.37
             |                   0.0000
-------------+----------------------------
Statistics   |
           N |          61            61
        r2_a |   .60516076     .99254275
------------------------------------------
                              legend: b/t/p
```



As we can see, the linear model (where Y1 is regressed only on X1) at first underestimates several values of y1, then overestimates, then goes back to underestimating them. By way of contrast, the quadratic model (where Y1 is regressed on X1 and $X1^2$) matches the observed data almost perfectly.

We can also graph the relationship using these Stata commands:

```
. scatter y1 x1 || lfit y1 x1 || qfit y1 x1
```



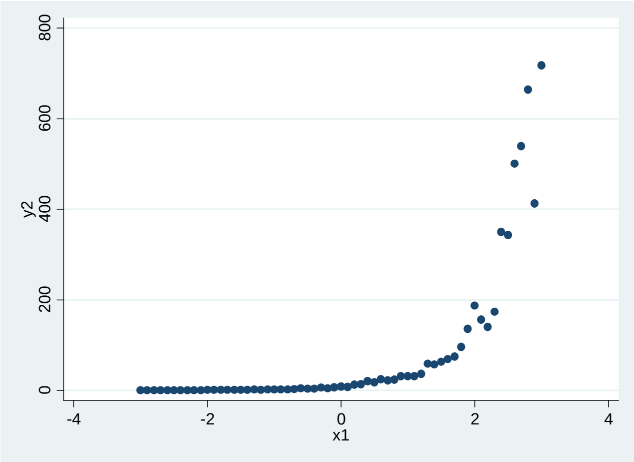
To estimate the quadratic model using the `regress` command (which gives the same estimates that `curvefit` did),

```
. reg y1 x1 c.x1#c.x1

      Source |       SS       df       MS              Number of obs =      61
-------------+------------------------------           F(  2,    58) = 3993.93
       Model |   308.65331     2  154.326655           Prob > F      =  0.0000
    Residual |  2.24113791    58  .038640309           R-squared     =  0.9928
-------------+------------------------------           Adj R-squared =  0.9925
       Total |  310.894448    60  5.18157413           Root MSE      =  .19657

------------------------------------------------------------------------------
          y1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |    1.00287   .0142947    70.16   0.000     .9742561    1.031484
             |
 c.x1#c.x1 |   .5028066   .0090808    55.37   0.000     .4846294    .5209838
             |
       _cons |   2.966442    .037761    78.56   0.000     2.890855    3.042029
------------------------------------------------------------------------------
```

Such a model might explain the link between political ideology and political activism: the more extreme one is in his or her political views (in either direction), the more likely he or she is to be politically active. Those in the middle of the road are least active. The tendency is more pronounced for right-wing ideologues than for left-wing.

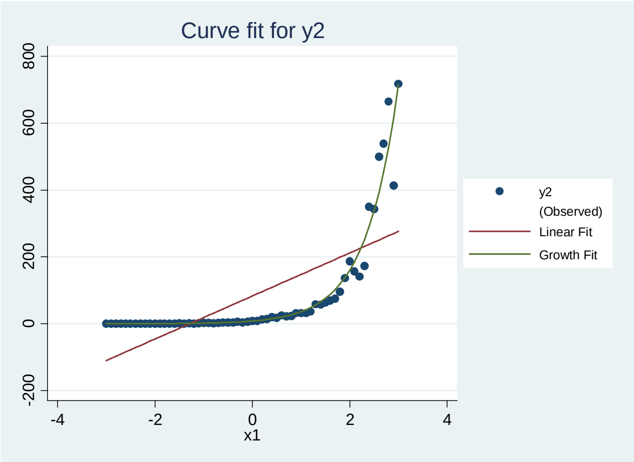
For Y2, the plot with X1 is

```
. ********** Part 2.
. * Plot of x1 with y2
. estimates clear
. scatter y2 x1
```



This suggests exponential growth. The points increase slowly at first, and then grow by larger and larger amounts. Plotting a linear and exponential model with `curvefit`,

```
. curvefit y2 x1, f(1 0)

Curve Estimation between y2 and x1

------------------------------------------
    Variable |    Linear         Growth
-------------+----------------------------
b0           |
       _cons |   83.005824      2.083302
             |        5.34          8.83
             |      0.0000        0.0000
-------------+----------------------------
b1           |
       _cons |   64.366035      1.496315
             |        7.29         17.34
             |      0.0000        0.0000
-------------+----------------------------
Statistics   |
           N |          61            61
        r2_a |   .46518882      .9519786
------------------------------------------
                             legend: b/t/p
```



As `curvefit` shows, if we just use a linear model with Y2 dependent, we will first underestimate the values of y2, then over-estimate them, then go back to underestimating them again.

We could estimate a model where we computed the log of y2 and regressed it on x. The potential problem with this approach is that the log of 0 is undefined; ergo, any cases with 0 (or for that matter negative) values will get dropped from the analysis. Further, most of us don't think in terms of logs of variables; we would rather see how X is related to the unlogged Y. It is therefore often better to estimate this model:

$$E(Y) = e^{(\alpha + \beta X)}$$

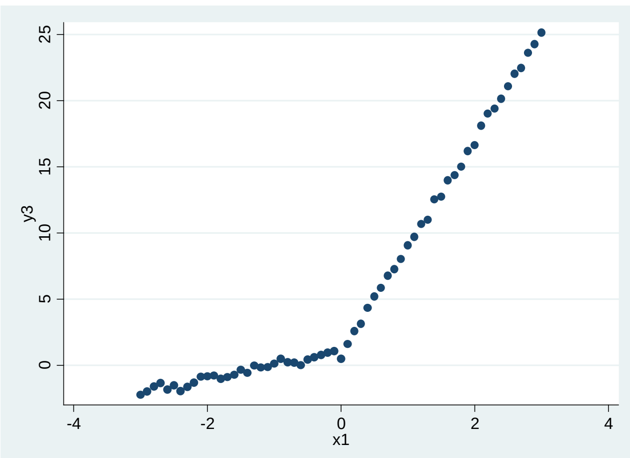
When you do this, Y itself can equal 0; all that is required is that its expected value be greater than zero. In Stata, we can estimate this as a *generalized linear model* with link log. The command and results are

```
. glm y2 x1, link(log)

Generalized linear models                          No. of obs      =        61
Optimization     : ML                              Residual df     =        59
                                                   Scale parameter =  1631.748
Deviance         =   96273.1221                    (1/df) Deviance =  1631.748
Pearson          =   96273.1221                    (1/df) Pearson  =  1631.748

Variance function: V(u) = 1                        [Gaussian]
Link function    : g(u) = ln(u)                    [Log]

                                                   AIC             =  10.26752
Log likelihood   = -311.1594035                    BIC             =  96030.58

------------------------------------------------------------------------------
             |                 OIM
          y2 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   1.496316   .0824864    18.14   0.000     1.334645    1.657986
       _cons |     2.0833   .2256518     9.23   0.000     1.641031     2.52557
------------------------------------------------------------------------------
```

Exponential/Growth models are good for variables such as income or sales that we expect to increase by percentage rather than absolute amounts. For example, we might predict that each dollar increase in cost would reduce sales by 5%.

For Y3, the plot is

```
. ********** Part 3.
. * Plot of x1 with y3.
. estimates clear
. scatter y3 x1
```



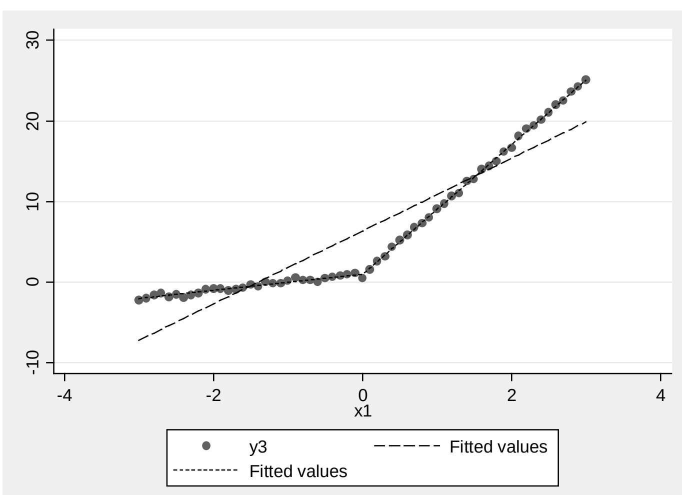
Here, there seem to be two pieces to the data. For X < 0, there is a small slope. Then, the slope becomes much greater. Ergo, a piecewise regression seems called for, and the `mkspline` command will make that possible:

```
. mkspline xle0 0 xgt0 = x1, marginal
. reg y3 x1
```

```
      Source |       SS       df       MS              Number of obs =      61
-------------+------------------------------           F(  1,    59) =  386.41
       Model |  3852.18882     1  3852.18882           Prob > F      =  0.0000
    Residual |   588.18379    59  9.96921678           R-squared     =  0.8675
-------------+------------------------------           Adj R-squared =  0.8653
       Total |  4440.37261    60  74.0062102           Root MSE      =  3.1574

------------------------------------------------------------------------------
          y3 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          x1 |   4.513444   .2296068    19.66   0.000     4.054001    4.972886
       _cons |   6.329963   .4042645    15.66   0.000     5.521032    7.138895
------------------------------------------------------------------------------

. predict linear
(option xb assumed; fitted values)
```

The linear model is a pretty good fit, but we can do better.

```
. reg y3 xle0 xgt0

      Source |       SS       df       MS              Number of obs =      61
-------------+------------------------------           F(  2,    58) =41949.15
       Model |  4437.30505     2  2218.65252           Prob > F      =  0.0000
    Residual |  3.06756763    58  .052889097           R-squared     =  0.9993
-------------+------------------------------           Adj R-squared =  0.9993
       Total |  4440.37261    60  74.0062102           Root MSE      =  .22998

------------------------------------------------------------------------------
          y3 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        xle0 |   .9967843   .0373837    26.66   0.000     .9219526    1.071616
        xgt0 |   7.033319   .0668686   105.18   0.000     6.899467    7.167171
       _cons |    .968499   .0588672    16.45   0.000     .8506636    1.086334
------------------------------------------------------------------------------

. predict spline
(option xb assumed; fitted values)
. scatter y3 x1 || line linear x1 || line spline x1, sort scheme(sj)
```

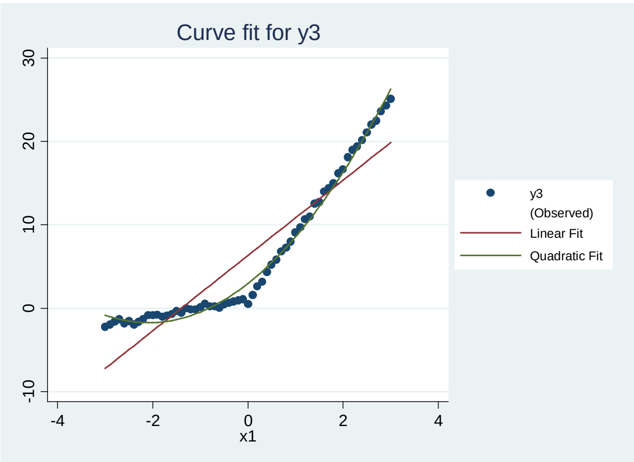


As the graph shows, if we just estimate a linear model, we will first underestimate y3, then over-estimate, then underestimate again.The fit of the piecewise model is near perfect in this case.

Substantive example: Years of college education may have much more impact on earnings than years of elementary education.

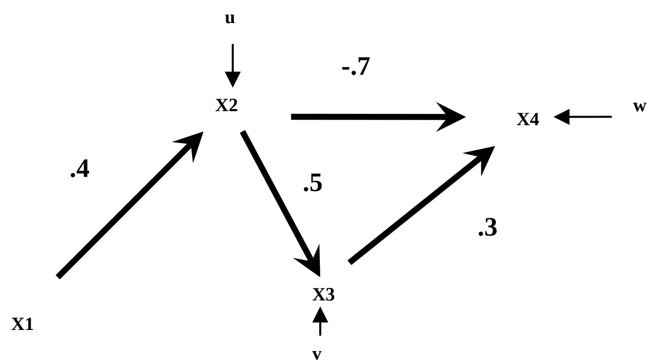
Finally, we present an alternative curvefit for Y3. Instead of doing piecewise regression, we fit a quadratic model:

```
. estimates clear
. curvefit y3 x1, f(1 4)

Curve Estimation between y3 and x1

------------------------------------------
    Variable |   Linear        Quadratic
-------------+----------------------------
b0           |
       _cons |   6.3299633     2.9758679
             |       15.66         18.74
             |      0.0000        0.0000
-------------+----------------------------
b1           |
       _cons |   4.5134436     4.5134436
             |       19.66         75.09
             |      0.0000        0.0000
-------------+----------------------------
b2           |
       _cons |                 1.0819662
             |                     28.33
             |                    0.0000
-------------+----------------------------
Statistics   |
           N |          61            61
        r2_a |   .86529216     .99076762
------------------------------------------
                             legend: b/t/p
```



As we see, the quadratic model also seems to provide a very good fit to the observed data. Unless relationships are extremely strong, plots of the data may reveal that nonlinearity is present, but won't necessarily make it obvious what the best solution is. Theory should guide you as you attempt to determine what solution is most appropriate.

Problem 2. A sociologist believes that the following model describes the relationships between X1, X2, X3 and X4. All variables are in standardized form. The hypothesized value of each path is included in the diagram.



a. Write out the structural equation for each endogenous variable, using both the names for the paths (e.g. β_{42}) and the estimated value of the path coefficient.

$$X_2 = \beta_{21} X_1 + u = .4X_1 + u$$
$$X_3 = \beta_{32} X_2 + v = .5X_2 + v$$
$$X_4 = \beta_{42} X_2 + \beta_{43} X_3 + w = -.7X2 + .3X3 + w$$

b. Part of the correlation matrix is shown below. Determine the complete correlation matrix. (Remember, variables are standardized. You can use either normal equations or Sewell Wright, but you might want to use both as a double-check.)

	x1	x2	x3	x4
x1	1.0000			
x2	0.4000	1.0000		
x3	?	?	1.0000	
x4	-0.2200	?	?	1.0000

Here is the complete correlation matrix:

	x1	x2	x3	x4
x1	1.0000			
x2	0.4000	1.0000		
x3	0.2000	0.5000	1.0000	
x4	-0.2200	-0.5500	-0.0500	1.0000

To confirm, using normal equations (in this case though, it may be easier just to look at the diagram and use Sewell Wright):

$$\rho_{31} = \beta_{31} + \beta_{21}\beta_{32} = 0 + .4 * .5 = .20$$
$$\rho_{32} = \beta_{32} + \beta_{31}\beta_{21} = .5 + 0 * .4 = .5$$
$$\rho_{42} = \beta_{42} + \beta_{32}\beta_{43} + \beta_{41}\beta_{21} + \beta_{43}\beta_{31}\beta_{21} = -.7 + .5 * .3 + 0 * .4 + .3 * 0 * .4 = -.55$$
$$\rho_{43} = \beta_{43} + \beta_{41}\beta_{31} + \beta_{42}\beta_{32} + \beta_{41}\beta_{21}\beta_{32} + \beta_{42}\beta_{21}\beta_{31} = .3 + 0 * 0 + -.7 * .5 + 0 * .4 * .5 + -.7 * .4 * 0 = -.05$$

c. (5 pts) Decompose the correlation between X3 and X4 into

- Correlation due to direct effects

.3

- Correlation due to indirect effects

0

- Correlation due to common causes

-.35

d. Suppose the above model is correct, but instead the researcher believed in and estimated the following model:

$$\mathbf{X3} \longrightarrow \mathbf{X4} \longleftarrow \mathbf{w}$$

What conclusions would the researcher likely draw? In particular, what would the researcher conclude about the effect of changes in X3 on X4? Why would he make these mistakes? Discuss the consequences of this mis-specification.

In the mis-specified model, the standardized coefficient will be the same as the bivariate correlation, -.05. The researcher will therefore conclude that increases in X3 lead to decreases in X4. However, in the correctly specified model, we see that the direct effect of X3 on X4 is .3, i.e. increases in X3 lead to increases in X4. By failing to take into account the common cause, X2, the research will not only mis-estimate the magnitude but also the direction of the effect of X3 on X4. If policy issues were involved, policy makers might do exactly the opposite of what they should do.

e. [Optional] Confirm your answer to 2b using Stata, i.e. create a pseudo-replication of the data using `corr2data` and then use one of the methods described in the notes for making sure that you can reproduce the estimates of the path coefficients given in the diagram.

We can use `pathreg` or `sem` (`pathreg` from UCLA must be installed),

```
. matrix input corr = (1,.4,.2,-.22\.4,1,.5,-.55\.2,.5,1,-.05\-.22,-.55,-.05,1)
. corr2data x1 x2 x3 x4, n(100) corr(corr)
(obs 100)
. pathreg (x2 x1) (x3 x1 x2) (x4 x1 x2 x3)

------------------------------------------------------------------------------
          x2 |      Coef.   Std. Err.      t    P>|t|                     Beta
-------------+----------------------------------------------------------------
          x1 |         .4    .092582     4.32   0.000                       .4
       _cons |   3.07e-09   .0921179     0.00   1.000                        .
------------------------------------------------------------------------------
                 n = 100  R2 = 0.1600  sqrt(1 - R2) = 0.9165

------------------------------------------------------------------------------
          x3 |      Coef.   Std. Err.      t    P>|t|                     Beta
-------------+----------------------------------------------------------------
          x1 |   4.27e-09   .0959412     0.00   1.000                 4.27e-09
          x2 |         .5   .0959412     5.21   0.000                       .5
       _cons |   1.36e-09   .0874908     0.00   1.000                        .
------------------------------------------------------------------------------
                 n = 100  R2 = 0.2500  sqrt(1 - R2) = 0.8660

------------------------------------------------------------------------------
          x4 |      Coef.   Std. Err.      t    P>|t|                     Beta
-------------+----------------------------------------------------------------
          x1 |  -8.76e-09   .0883883    -0.00   1.000                -8.76e-09
          x2 |        -.7         .1    -7.00   0.000                      -.7
          x3 |         .3   .0935414     3.21   0.002                       .3
       _cons |  -8.66e-09   .0806032    -0.00   1.000                        .
------------------------------------------------------------------------------
                 n = 100  R2 = 0.3700  sqrt(1 - R2) = 0.7937

. sem (x2 <- x1) (x3 <- x1 x2) (x4 <- x1 x2 x3)

Endogenous variables

Observed:  x2 x3 x4

Exogenous variables

Observed:  x1

Fitting target model:

Structural equation model                       Number of obs      =       100
Estimation method  = ml
Log likelihood     =  -519.3618
```

```
------------------------------------------------------------------------------
             |                 OIM
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Structural   |
  x2 <-      |
          x1 |         .4   .0916515     4.36   0.000     .2203663    .5796337
       _cons |   3.07e-09   .0911921     0.00   1.000    -.1787332    .1787332
  -----------+----------------------------------------------------------------
  x3 <-      |
          x2 |         .5   .0944911     5.29   0.000     .3148008    .6851992
          x1 |   4.27e-09   .0944911     0.00   1.000    -.1851992    .1851992
       _cons |   1.36e-09   .0861684     0.00   1.000     -.168887     .168887
  -----------+----------------------------------------------------------------
  x4 <-      |
          x2 |        -.7   .0979796    -7.14   0.000    -.8920364   -.5079635
          x3 |         .3   .0916515     3.27   0.001     .1203663    .4796337
          x1 |  -8.76e-09   .0866025    -0.00   1.000    -.1697379    .1697379
       _cons |  -8.66e-09   .0789747    -0.00   1.000    -.1547875    .1547875
-------------+----------------------------------------------------------------
Variance     |
        e.x2 |      .8316    .117606                      .6302842    1.097217
        e.x3 |      .7425   .1050054                      .5627537    .9796581
        e.x4 |      .6237   .0882045                      .4727131    .8229128
------------------------------------------------------------------------------
LR test of model vs. saturated: chi2(0)   =      0.00, Prob > chi2 =      .
```

The following post-estimation command after `sem` can confirm our estimates of direct, indirect and total effects (but not correlation due to common cause, alas):

```
Direct effects
------------------------------------------------------------------------------
             |                 OIM
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Structural   |
  x2 <-      |
          x1 |         .4   .0916515     4.36   0.000     .2203663    .5796337
  -----------+----------------------------------------------------------------
  x3 <-      |
          x2 |         .5   .0944911     5.29   0.000     .3148008    .6851992
          x1 |   4.27e-09   .0944911     0.00   1.000    -.1851992    .1851992
  -----------+----------------------------------------------------------------
  x4 <-      |
          x2 |        -.7   .0979796    -7.14   0.000    -.8920364   -.5079635
          x3 |         .3   .0916515     3.27   0.001     .1203663    .4796337
          x1 |  -8.76e-09   .0866025    -0.00   1.000    -.1697379    .1697379
------------------------------------------------------------------------------
```

```
Indirect effects
------------------------------------------------------------------------------
             |                 OIM
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Structural   |
  x2 <-      |
          x1 |          0  (no path)
  -----------+----------------------------------------------------------------
  x3 <-      |
          x2 |          0  (no path)
          x1 |         .2   .0594018     3.37   0.001     .0835746    .3164253
  -----------+----------------------------------------------------------------
  x4 <-      |
          x2 |        .15   .0283473     5.29   0.000     .0944402    .2055598
          x3 |          0  (no path)
          x1 |       -.22    .066453    -3.31   0.001    -.3502455   -.0897545
------------------------------------------------------------------------------


Total effects
------------------------------------------------------------------------------
             |                 OIM
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Structural   |
  x2 <-      |
          x1 |         .4   .0916515     4.36   0.000     .2203663    .5796337
  -----------+----------------------------------------------------------------
  x3 <-      |
          x2 |         .5   .0944911     5.29   0.000     .3148008    .6851992
          x1 |         .2   .0979796     2.04   0.041     .0079635    .3920365
  -----------+----------------------------------------------------------------
  x4 <-      |
          x2 |       -.55   .1019979    -5.39   0.000    -.7499122   -.3500878
          x3 |         .3   .0916515     3.27   0.001     .1203663    .4796337
          x1 |       -.22     .09755    -2.26   0.024    -.4111945   -.0288055
------------------------------------------------------------------------------
```

The complete Stata code for problem 2 is

```
version 12.1
* Homework 7, Path analysis problem
clear all
matrix input corr = (1,.4,.2,-.22\.4,1,.5,-.55\.2,.5,1,-.05\-.22,-.55,-.05,1)
corr2data x1 x2 x3 x4, n(100) corr(corr)
corr
pathreg (x2 x1) (x3 x1 x2) (x4 x1 x2 x3)
sem (x2 <- x1) (x3 <- x1 x2) (x4 <- x1 x2 x3)
estat teffects
```