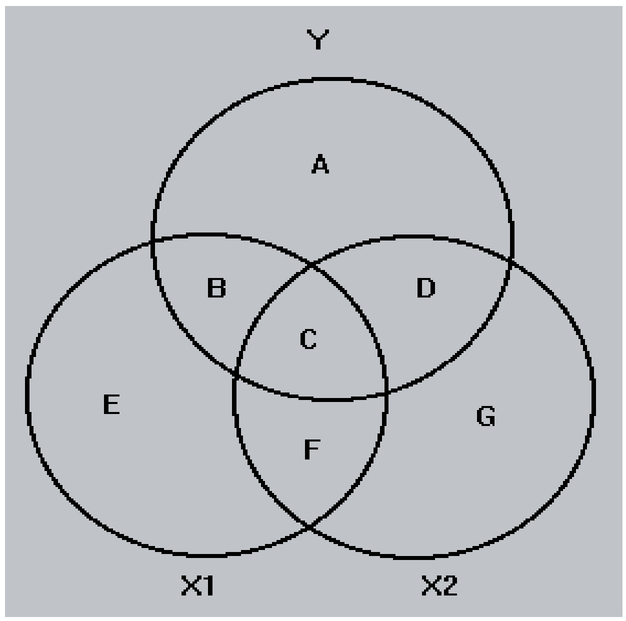
Supplemental notes on Semipartial Correlations

> This discussion borrows heavily from <u>Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences</u>, by Jacob and Patricia Cohen (1975 edition; there is also an updated 2003 edition now).

When I presented the following diagram, I pointed out that this was just one example of the many ways that the Xs and Ys could be interrelated:



I also presented the output for this specific empirical example:

Correlations

		INCOME	EDUC	JOBEXP
Pearson Correlation	INCOME	1.000	.846	.268
	EDUC	.846	1.000	-.107
	JOBEXP	.268	-.107	1.000

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.919[a]	.845	.827	4.07431

a. Predictors: (Constant), JOBEXP, EDUC

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95% Confidence Interval for B		Correlations			Collinearity Statistics	
		B	Std. Error	Beta			Lower Bound	Upper Bound	Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	-7.097	3.626		-1.957	.067	-14.748	.554					
	EDUC	1.933	.210	.884	9.209	.000	1.490	2.376	.846	.913	.879	.989	1.012
	JOBEXP	.649	.172	.362	3.772	.002	.286	1.013	.268	.675	.360	.989	1.012

a. Dependent Variable: INCOME

However, in February 2004, Eugene Paik, a graduate student at UNLV, emailed me and pointed out that the diagram didn't seem to correspond to the empirical example! (Later, Catherine Liu, graduate student at Notre Dame, made the same observation.) Paik's argument was as follows:

Given

- **Y is the dependent variable.**
- **X1 and X2 are independent variables.**
- $\mathbf{R}^2_{y12} = \mathbf{B} + \mathbf{C} + \mathbf{D}$
- $\mathbf{r}^2_{y(1.2)} = \mathbf{B}$
- $\mathbf{r}^2_{y(2.1)} = \mathbf{D}$

Then

- $\mathbf{r}^2_{y(1.2)} + \mathbf{r}^2_{y(2.1)} \leq \mathbf{R}^2_{y12}$ **for all cases.**
- **In other words,** $\mathbf{C} \geq 0$
- **In other words, the sum of the squares of all semi-partial correlations cannot exceed R^2.**

Because

- **In term of the variance that each semi-partial correlation accounts for in Y, they are mutually exclusive by definition.**
- **In other words, the areas corresponding to semi-partial correlations do not overlap in the Ballantine diagram.**

However

- **In the example provided, this does not hold.**
- **The sum of squared semi-partial (part) correlations from the SPFF example is $0.879^2 + 0.360^2$, which is around 0.9022.**
- **But R^2 (0.845) is smaller!!!**
- **If you calculate the area C, you get around -0.0591. That's negative 0.0591.**

What does it mean for the sum of squared semi-partial correlations to exceed R^2? What does it mean for the area of C to be negative?

Incidentally, I've seen even more extreme examples where C is largely negative (e.g., -0.41).

To summarize, the diagram implies that $R^2_{Y12} = B + C + D$, $sr^2_1 = B$, $sr^2_2 = D$, so

$$R^2_{Y12} \geq sr^2_1 + sr^2_2 \text{ because } B + C + D \geq B + D.$$

However, in the actual example, $R^2_{Y12} = .845$, $sr^2_1 = .773$, $sr^2_2 = .1296$, hence

$$R^2_{Y12} < sr^2_1 + sr^2_2 \text{ because } .845 < .902.$$

Paik is right; while I think the diagram is very useful and works for many situations, it doesn't accurately describe the specific example I am using. Here is the response I sent to Paik:

> The example I use (which may be a good reason for not using it!) is an example of what Cohen and Cohen call Cooperative Suppression. Note that
>
> - Educ and Jobexp are negatively correlated with each other (which makes sense; get more of one, you tend to get less of the other)
> - Nonetheless, both have positive correlations and effects on income (which again makes sense; the more education and job experience you have, the more you can expect to make)
> - As a result, the semipartial (part) correlations are actually larger than the zero-order correlations are.
>
> In the attached excerpt from the 1975 edition of their book (which I think explains this more clearly than the 2003 edition does) they show how this can lead to the sort of situation you describe, i.e. the sum of the squared semipartials is greater than the R^2. (Their whole chapter is worth reading if you can get a copy of it.)
>
> When cooperative suppression is present, the ballantine presentation breaks down a bit, because, as you say, you can't draw a negative area! Indeed, in their discussion, Cohen and Cohen present several diagrams, but they don't present one for cooperative suppression.

The diagram presented is fine when, say, X1, X2 and Y are all positively correlated; but that isn't the case in the current empirical example.

We will discuss the idea of suppression further during Stats II. Meanwhile, the attached page from Cohen and Cohen (1975) briefly discusses the idea.

3.4.6 Cooperative Suppression: $r_{12} < 0$

In this never ending search for high Rs, no circumstance is more attractive to the researcher (although rarely attained) than one in which IVs which correlate positively with Y correlate negatively with each other (or, equivalently, the reverse). r_{12}, being negative, involves a portion of the variance in the IVs all of which is irrelevant to Y; thus, when each variable is partialled from the other, all indices of relationship with Y are enhanced.

For example, imagine that a Director of Personnel is establishing a procedure for selecting sales personnel from among job applicants. A sample of current salesmen is drawn, and each person is rated for overall success in sales performance. These ratings constitute the "criterion" or dependent variable, Y. A series of interviews leads the researcher to suspect that two major components of sales success are social aggressiveness and habits and skills with regard to record keeping. Measures of these two characteristics are devised and administered to the sample of salesmen. The correlation between the measure of social aggressiveness (X_1) and sales success (Y) is found to be .29, the correlation between record keeping (X_2) and Y is .24, and $r_{12} = -.30$, indicating an overall tendency for those high on social aggressiveness to be relatively low on record keeping.

The semipartial correlations are

$$sr_1 = \frac{.29 - .24(-.30)}{\sqrt{1 - .30^2}} = \frac{.29 + .0720}{.9539} = .379,$$

$$sr_2 = \frac{.24 - .29(-.30)}{\sqrt{1 - .30^2}} = \frac{.24 + .0870}{.9539} = .343,$$

both larger than their respective validity coefficients. $R^2_{Y \cdot 12}$ is found by Eq. (3.3.1) to be

$$R^2_{Y \cdot 12} = \frac{.29^2 + .24^2 - 2(.29)(.24)(-.30)}{1 - (-.30)^2} = .2016,$$

and we note that

$$\begin{aligned} r^2_{Y1} + r^2_{Y2} &< R^2_{Y \cdot 12} < sr^2_1 + sr^2_2 \\ .0841 + .0576 &< .2016 < .1440 + .1175. \end{aligned}$$

Thus, the independent variables are mutually enhancing under conditions of "cooperative" suppression, and each variable accounts for a *larger* proportion of the Y variance in the presence of the other than it does alone.

Computing the βs we find that $\beta_1 = .398$ and $\beta_2 = .359$, both of which retain the sign and exceed in magnitude their respective r_{Yi}, .29 and .24.

Finally, it is important to note that all three kinds of suppression—classical, net, and cooperative—are not frequently found in behavioral science studies. The detailed presentation here is in the interest of enabling the researcher to recognize them when they do occur, and for their value as quasiparadoxical curiosities. A β_i coefficient which falls outside the limits defined by r_{Yi} and 0 signals the presence of suppression. If the X_i in question has a zero (in practice, very small) correlation with Y, the situation is one of classical suppression. If its β_i is of opposite sign from its r_{Yi}, it is serving as a net suppressor. If its β_i exceeds its r_{Yi} and is of the same sign, cooperative suppression is indicated.