

Two sample tests – Special Topics; Using SPSS

For case IV, Matched pairs, we gave an example of husbands and wives and their influence on family decision-making. We are going to take another look at that example and do the following:

I. We originally solved the problem by computing D , and then finding the sample variance of D . Now we'll show how s_D could be calculated by using the sample variances and covariances of H and W .

II. We'll see what would happen if we erroneously treated this problem as though it fell under two sample tests, case II or III. That is, we'll see what would happen if we acted as though the samples of husbands and wives were independent of each other, when they really weren't.

III. We'll see how SPSS could be used to estimate models II, III, and IV. (As far as I know, SPSS can't do Case I.)

IV. Also, as far as I can tell, SPSS doesn't provide a direct test of Model V. So, using our model V example from the last handout, we'll see what happens when Model II is used in a large sample when Model V is technically called for.

I. *Solution using sample variances and covariances.* Let A and B be any two variables. Then, the sample covariance of A and B is

$$s_{AB} = \frac{\sum (A_i - \bar{A})(B_i - \bar{B})}{N - 1} = \frac{\sum A_i B_i - N * \bar{A} * \bar{B}}{N - 1} = \frac{\sum A_i B_i - (\sum A_i \sum B_i) / N}{N - 1}$$

Note that, when $A = B$ (e.g. $A = B = X_1$) then the formula for the sample covariance is the same as the formula for the sample variance. Remember, the covariance of any variable with itself is equal to the variance of the variable. Hence, s_{AA} is an alternative notation for s_A^2 .

Let us now take another look at the family decision-making example.

Pair	H	W	$(H - \bar{H})^2$	$(W - \bar{W})^2$	$(H - \bar{H})(W - \bar{W})$
1	26	30	19.141	12.250	-15.313
2	28	29	5.641	6.250	-5.938
3	28	28	5.641	2.250	-3.563
4	29	27	1.891	0.250	-0.688
5	30	26	0.141	0.250	0.188
6	31	25	0.391	2.250	-0.938
7	34	24	13.141	6.250	-9.063
8	37	23	43.891	12.250	-23.188
Σ	243	212	89.875	42.000	-58.5

Then,

$$\bar{H} = 243/8 = 30.375$$

$$\bar{W} = 212/8 = 26.5$$

$$\bar{D} = \bar{H} - \bar{W} = 30.375 - 26.5 = 3.875$$

$$s_{HH} = s_1^2 = \frac{\sum(H - \bar{H})^2}{N - 1} = \frac{89.875}{7} = 12.839, s_1 = 3.583$$

$$s_{WW} = s_2^2 = \frac{\sum(W - \bar{W})^2}{N - 1} = \frac{42}{7} = 6, s_2 = 2.449$$

$$s_{HW} = s_{12} = \frac{\sum(H - \bar{H})(W - \bar{W})}{N - 1} = \frac{-58.5}{7} = -8.357$$

$$s_D^2 = s_1^2 + s_2^2 - 2s_{12} = 12.839 + 6 - (2 * -8.357) = 35.553$$

$$s_D = \sqrt{35.553} = 5.963$$

Note that these are the same values that we got when we computed s_D^2 and s_D before.

Or, if you prefer, you can work it this way:

Pair #	H score	W score	H ²	W ²	HW
1	26	30	676	900	780
2	28	29	784	841	812
3	28	28	784	784	784
4	29	27	841	729	783
5	30	26	900	676	780
6	31	25	961	625	775
7	34	24	1156	576	816
8	37	23	1369	529	851
Σ	243	212	7,471	5,660	6,381

Hence, we get

$$s_{HH} = \frac{\sum H_i H_i - (\sum H_i \sum H_i) / N}{N - 1} = \frac{\sum H_i^2 - (\sum H_i)^2 / N}{N - 1} = \frac{7,471 - 243^2 / 8}{7} = 12.839,$$

$$s_{WW} = \frac{\sum W_i W_i - (\sum W_i \sum W_i) / N}{N - 1} = \frac{\sum W_i^2 - (\sum W_i)^2 / N}{N - 1} = \frac{5,660 - 212^2 / 8}{7} = 6,$$

$$s_{HW} = \frac{\sum H_i W_i - (\sum H_i \sum W_i) / N}{N - 1} = \frac{6,381 - (243 * 212) / 8}{7} = -8.357$$

Now, to compute s_D^2 and s_D ,

$$s_D^2 = s_{HH} + s_{WW} - 2s_{HW} = 12.839 + 6 - (2 * -8.357) = 35.553$$

$$s_D = \sqrt{s_D^2} = \sqrt{35.553} = 5.963$$

II. *Mistaken use of two sample tests, cases II and III.* Let us now see what would happen if we erroneously treated the husband and wife samples as though they were independent. For case II, σ 's unknown but assumed equal, we would proceed as follows:

Step 1:

$$\begin{array}{ll} H_0: & E(H) - E(W) = 0 \quad \text{or, } E(H) = E(W) \\ H_A: & E(H) - E(W) \neq 0 \quad \text{or, } E(H) \neq E(W) \end{array}$$

Step 2. The (in)appropriate test statistic is

$$t = \frac{\hat{\mu}_1 - \hat{\mu}_2 - (\mu_1 - \mu_2)_0}{\sqrt{\left(\frac{(N_1 - 1)s_1^2 + (N_2 - 1)s_2^2}{N_1 + N_2 - 2}\right)\left(\frac{N_1 + N_2}{N_1 N_2}\right)}} = \frac{\hat{\mu}_1 - \hat{\mu}_2}{\sqrt{\left(\frac{7s_1^2 + 7s_2^2}{8 + 8 - 2}\right)\left(\frac{8 + 8}{8 * 8}\right)}} = \frac{\hat{\mu}_1 - \hat{\mu}_2}{1.5346}$$

(The final equality, of course, could not be determined until after we knew what the two sample variances were.)

Step 3. Since $n_1 + n_2 - 2 = 14$, for $\alpha = .05$ we will accept H_0 if $-2.145 \leq T_{14} \leq 2.145$.

Step 4.

$$t = \frac{\hat{\mu}_1 - \hat{\mu}_2}{1.5346} = \frac{30.375 - 26.5}{1.5346} = 2.525$$

Step 5. We should reject H_0 , the computed T value is outside the acceptance region.

Let us now treat this problem as though it fell under Case III, σ 's unknown and not assumed equal. Recall that, when $N_1 = N_2$, as is the case here (i.e. the group sizes are equal) Case II and Case III both yield the same value for the computed test statistic, i.e. 2525. The degrees of freedom for the case III t test are

$$\begin{aligned} v &= \frac{\left(\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}\right)^2}{\left(\frac{s_1^2}{N_1}\right)^2\left(\frac{1}{N_1 - 1}\right) + \left(\frac{s_2^2}{N_2}\right)^2\left(\frac{1}{N_2 - 1}\right)} \\ &= \frac{\left(\frac{12.839}{8} + \frac{6}{8}\right)^2}{\left(\frac{12.839}{8}\right)^2\left(\frac{1}{8 - 1}\right) + \left(\frac{6}{8}\right)^2\left(\frac{1}{8 - 1}\right)} \\ &= 12.37 \end{aligned}$$

For $v = 12$, for $\alpha = .05$ we will accept H_0 if $-2.179 \leq T_{12} \leq 2.179$. Since $t = 2.525$ we still reject.

COMMENTS.

When we erroneously treat the two samples as independent, we reach the wrong conclusion, i.e. we reject H_0 when we previously saw that we should not reject it. This occurs for several reasons:

a. The degrees of freedom are wrong when we treat the samples as independent (14 d.f., as opposed to the correct value of 7 d.f.). This changes the acceptance region.

b. More critically, the estimated standard error of the difference is wrong. Under the incorrect specification, the estimated standard error is 1.5346 (see above). Under the correct specification, the estimated standard error is 2.1081 (see earlier example for 2 sample tests, case IV). These differences cause the test statistic to have a greater value under the incorrect specification.

Problems like this can occur whenever two samples are treated as independent when they really aren't. It is also possible, and perhaps more common, for the opposite to happen; that is, if the two variables positively covary (instead of negatively, as is the case here) the matched pairs test statistic value could be too low, causing you to not reject the null when you should.

III. *SPSS solution for Cases II, III and IV.* We will now show how this problem could be handled using SPSS-PC. The strategy is as follows:

1. For cases II and III, we enter each husband and wife score as though each belonged to a different case; that is, we enter two cases for each pair of data, giving us 16 cases altogether. For each case, the variables are the gender of the respondent (male or female) and the respondent's score.

2. We then run a group T-test. The dependent variable, SCORE, is broken down by the two groups, male and female. SPSS reports the results for both cases II and III. SPSS's "Equal variances assumed" corresponds to two sample tests, Case II, while SPSS's "Equal variances not assumed" corresponds to case III. Earlier versions of SPSS used the somewhat more cryptic labeling of "pooled variance estimate" for case II and "separate variance estimate" for case III. After reading in the data, the key command is

```
T-TEST /GROUPS SEX (1,2) /VARIABLES SCORE.
```

3. To estimate Case IV, matched pairs, two variables are read in for each of the eight cases: the husband's score (HSCORE) and the wife's score (WSCORE). We then tell SPSS to do a matched pairs t-test. After reading in the data, the key command is

```
T-TEST /PAIRS HScore WScore.
```

4. Note that all results are the same as what we have previously computed.

Here are the SPSS control cards for estimating cases II and III:

```
* First, we will (erroneously) treat these as 2 independent
* samples. SPSS will give us the results for 2 sample
* tests, cases II and III. 'Equal Variances assumed'
* corresponds to case II (sigma 1 = sigma 2 = sigma),
* 'Equal variances not assumed' corresponds to case III
* (sigma 1 does not equal sigma 2). Note that SPSS uses a
* different formula for Case III d.f. than I have been using.
* I do not know where their formula comes from or if it can be
* considered correct.
DATA LIST FREE / Sex Score.
BEGIN DATA.
1 26
2 30
1 28
```

2 29
 1 28
 2 28
 1 29
 2 27
 1 30
 2 26
 1 31
 2 25
 1 34
 2 24
 1 37
 2 23

END DATA.

VARIABLE LABELS SEX 'Sex of Respondent' SCORE 'Respondents score'.

VALUE LABELS SEX 1 'Male' 2 'Female'.

T-TEST /GROUPS SEX (1,2) /VARIABLES SCORE.

Here is part of the SPSS output:

T-Test

Group Statistics

	SEX Sex of Respondent	N	Mean	Std. Deviation	Std. Error Mean
SCORE	1.00 Male	8	30.3750	3.58319	1.26685
Respondents score	2.00 Female	8	26.5000	2.44949	.86603

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
SCORE	Equal variances assumed	.710	.414	2.525	14	.024	3.8750	1.53457	.58367	7.16633
Respondents score	Equal variances not assumed			2.525	12.370	.026	3.8750	1.53457	.54251	7.20749

Here are the SPSS control cards for estimating Case IV, Matched Pairs:

```
* Now we will treat this as a matched pairs problem.  SPSS will
* give us the results for 2 sample tests, case 4.
DATA LIST FREE / HScore Wscore.
BEGIN DATA.
26 30
28 29
28 28
29 27
30 26
31 25
34 24
37 23
END DATA.
VARIABLE LABELS HScore 'Husbands score' WScore 'Wifes score'.
T-TEST /PAIRS HScore WScore.
```

Here is part of the SPSS output:

T-Test

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	HSCORE Husbands score	30.3750	8	3.58319	1.26685
	WSCORE Wifes score	26.5000	8	2.44949	.86603

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	HSCORE Husbands score - WSCORE Wifes score	3.8750	5.96268	2.10813	-1.1099	8.8599	1.838	7	.109

IV. Finally, as far as I can tell, SPSS does not directly estimate Model V, $p_1 = p_2$. But, in a large sample, Model II tends to serve as a pretty good approximation. Recall this problem from the last handout: Two groups, A and B, each consist of 100 randomly assigned people who have a disease. One serum is given to Group A and a different serum is given to Group B; otherwise, the two groups are treated identically. It is found that in groups A and B, 75 and 65 people, respectively, recover from the disease. For the test statistic, we got

$$z = \frac{\frac{X_1}{100} - \frac{X_2}{100}}{\sqrt{\left(\frac{200}{10,000}\right)\left(\frac{X_1 + X_2}{200}\right)\left(1 - \frac{X_1 + X_2}{200}\right)}} = \frac{\frac{75}{100} - \frac{65}{100}}{\sqrt{\left(\frac{200}{10,000}\right)\left(\frac{75 + 65}{200}\right)\left(1 - \frac{75 + 65}{200}\right)}} = 1.543$$

and for the the approximate 95% c.i. we got

$$\hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{N_1} + \frac{\hat{p}_2 \hat{q}_2}{N_2}} = .75 - .65 \pm 1.96 \sqrt{\frac{.75 * .25}{100} + \frac{.65 * .35}{100}}, i.e.$$

$$-.026 \leq p_1 - p_2 \leq .226$$

Now, if we set this up as though this were a Model II problem, σ 's unknown but assumed equal, the SPSS cards are

* Finally, using different data, we'll show what happens when Model II is used
 * with a large data set when Model V is technically called for. Note that results
 * are virtually identical to what we got when we hand-calculated the results
 * for case V. Later, we'll show another and more exact way to get SPSS to
 * handle case V.

```
DATA LIST FREE / Group Recover Freq.
BEGIN DATA.
1 1 75
1 0 25
2 1 65
2 0 35
END DATA.
Value labels Group 1 "Group A" 2 "Group B"/
Recover 1 "Recovered" 0 "Didn't Recover"/.
Weight by Freq.
```

```
T-TEST
GROUPS=group(1 2)
/MISSING=ANALYSIS
/VARIABLES=recover
/CRITERIA=CIN(.95) .
```


The output is

T-Test

Group Statistics

	GROUP	N	Mean	Std. Deviation	Std. Error Mean
RECOVER	1.00 Group A	100	.7500	.43519	.04352
	2.00 Group B	100	.6500	.47937	.04794

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
									Lower	Upper
RECOVER	Equal variances assumed	9.408	.002	1.545	198	.124	.1000	.06475	-.02768	.22768
	Equal variances not assumed			1.545	196.177	.124	.1000	.06475	-.02769	.22769

Note that the values of the test statistic and the confidence interval are virtually identical to what we got before. Later, we'll show other ways in which you can approximate Model V even though SPSS doesn't seem to have a direct routine for it.