Economics 43535: Assignment 2 Key
(due Wednesday, March 19)

Professor Jensen

Spring 2014

1. Assume the market value of a pine forest as a function its age t is given by

$$V = 1000 + 10t - 5t^2$$

so that its rate of change over time is

$$dV/dt = 10 - 10t.$$

Also assume the interest rate is R = 0.1, the standing value of the forest is 300, the harvest cost is 500, and the value of the land is 3000.

Determine the optimal rotation t* that maximizes the net benefits of harvesting this forest.

Note that the marginal benefit and cost of waiting are

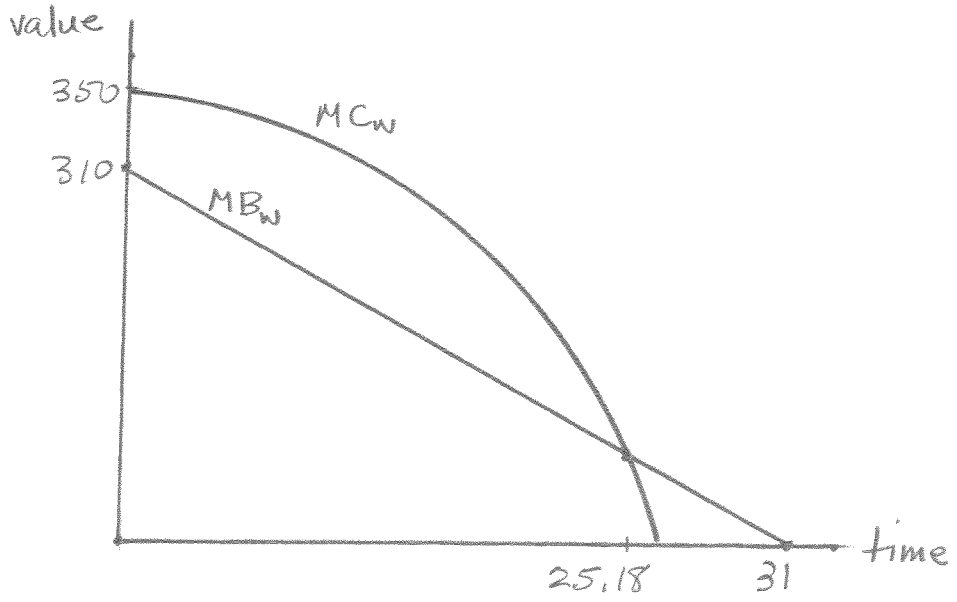
$$\mathbf{MB_W = dV/dt + SV = 10 - 10t + 300 = 310 - 10t}$$

and

$$\mathbf{MC_W = R(V - HC + LV) = .1(1000 + 10t - 5t^2 - 500 + 3000) = 350 + t - (.5)\, t^2.}$$

Setting $MB_W = MC_W$ gives $t^2 - 22t - 80 = 0$, which has roots t = 25.18 and t = – 3.18. The positive root t = 25.18 seems to make more sense as the correct answer, but it is not.

Notice that at t = 0, $MB_W = 310 < MC_W = 350$. This implies that we should not wait at all, but harvest immediately, t* = 0. If we wait one year, $MB_W = 300$ and $MC_W = 350.5$, so we lose 50.5. Waiting longer means even greater losses. You can tell this by looking at the graph, because (just as with MR and MC in a monopoly), we need MB_W to cut MC_W from above for a maximum. But MB_W cuts MC_W from below at t = 25.18, so that rotation actually minimizes profit. The correct answer is the "corner solution" at t* = 0.



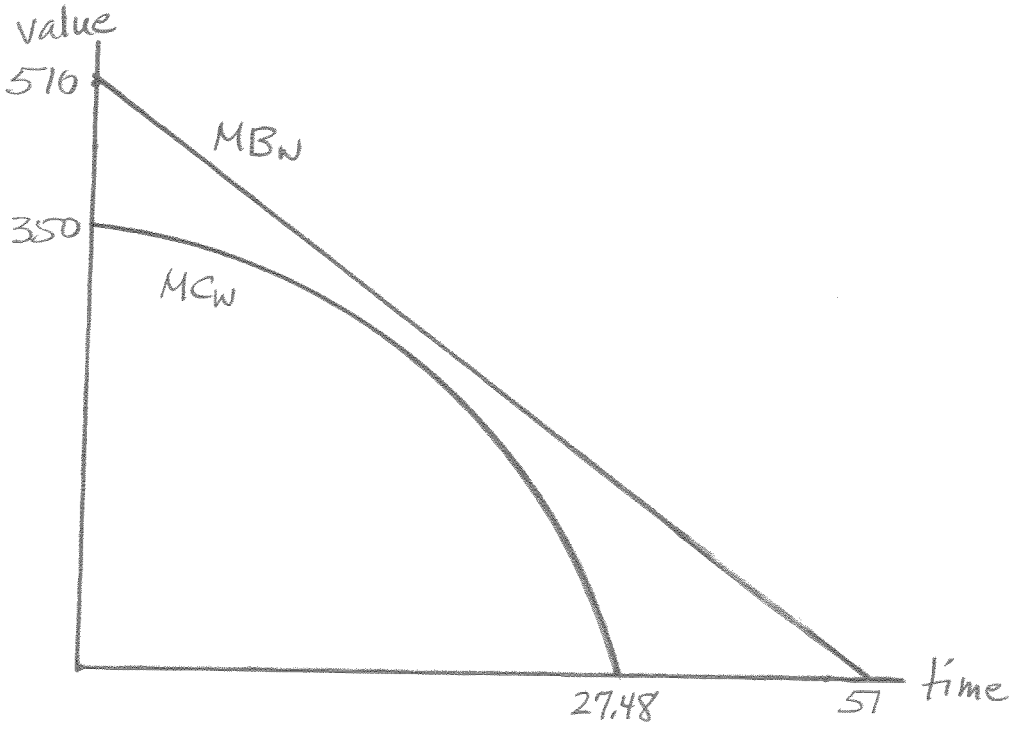
2. Suppose the firm that owns the forest hires an environmental economist and asks her to use state-of-the-art techniques to estimate the standing value of the forest. How does the optimal rotation change if her estimate of the standing value is 500? (HINT: It might be useful to graph the marginal benefit and marginal cost functions in this case.)

Note that the marginal benefit of waiting is now

$$MB_W = dV/dt + SV = 10 - 10t + 500 = 510 - 10t$$

and the marginal cost of waiting remains the same.

Setting $MB_W = MC_W$ gives $t^2 - 22t + 320 = 0$, which has imaginary roots. This seems strange until you draw the graph of MB_W and MC_W, as below, and see that the MB_W lies above the MC_W for all dates where MB_W is positive. The correct answer is never harvest (or $t^* = \infty$).



3. Provide an economic interpretation for the result found in #2 (i.e., what does an increase in ecological value do to the optimal harvesting rotation, and why?).

Notice from the graph that the marginal cost of waiting is less than the marginal benefit of waiting in #2 whenever both are positive (that is, MB_W lies above MC_W for all dates where MB_W is positive). In this case, the standing value of the forest is so high that the owners should never harvest it. This makes more sense for an "old growth" forest like the redwoods or sequoias, which have substantial ecological and passive use values, than pine, but you get the idea: large standing value can imply the best policy is never harvest.