Economics 70361: Problem Set 3
(due Wednesday, October 2)

Fall 2013 Professor Jensen

1. Consider a linear city with consumers uniformly distributed on the unit interval [0,1], where x denotes a location in [0,1], firm 1 is located at the left end of the city, x=0, and firm 2 is located at the right end of the city, x=1. Each consumer buys one unit of the good, but must travel to either firm 1 or firm 2 in order to buy. Transportation costs are quadratic in distance. That is, for t>0, a consumer at x pays tx^2 to travel to firm 1 and $t(1-x)^2$ to travel to firm 2. Thus, if p_1 is the price charged by firm 1, a consumer at x pays p_1+tx^2 to buy the good from firm 1 and $p_2+t(1-x)^2$ to buy the good from firm 2. Firm 1's constant average cost of production is $c_1>0$ and firm 2's is $c_2>0$. The firms compete by setting prices.
a. Determine the Nash equilibrium prices, demands, market shares, and profits for each firm.
b. Determine how these vary as functions of c_1 and c_2.
c. Under what conditions is firm 1's market share larger?

2. Now consider the linear city model with consumers uniformly distributed on [0,1]. Denote a location in [0,1] by x. Firm 1 is located at $x_1 \in [0,1]$ and firm 2 is located at $x_2 \in [0,1]$. Each consumer buys one unit of the good, but must travel to either firm 1 or firm 2 in order to buy. Transportation costs are t>0 per unit of distance, so if p_i is the price charged by firm i, a consumer who travels distance y to get to firm i pays p_i+ty to buy the good. The firms play the following sequential entry game. Firm 1 moves first, choosing its location x_1. This choice is revealed to all consumers and firm 2, which then chooses its location x_2. This choice is revealed to all consumers and firm 1. The firms then choose their prices simultaneously. Each firm's average cost of production is c>0.
a. Determine the subgame perfect equilibrium of this game.
b. Compare the prices, outputs, and profits of the firms in equilibrium.
c. Is there a first-mover advantage or second-mover advantage in this problem? Explain.

3. Consider the following market for vertically differentiated products. Consumers are uniformly distributed on the unit interval [0,1], where a value $\theta \in [0,1]$ represents the taste (or income) of the consumer at θ. Each consumer's utility is $U=\theta s-p$ if he/she buys one unit of a good of quality s at price p, and U=0 if he/she buys no good of any quality. The constant average cost of producing a good of quality s is cs^2, where c>0 is a constant. That is, the total cost of producing q units of quality s is $(cs^2)q$.
a. Determine the price p_m and quality s_m that maximize profit for a monopolist operating in this market. Is the market covered?
b. Suppose another firm considers entry into this market. That is, given the monopolist's choice of quality s_m in part a, the entrant chooses a quality s_e for its good. The firms the compete in prices. Determine the entrants quality choice and the Nash equilibrium prices, corresponding demands, market shares, and profits for each firm. Is the market covered?