Economics 70361: Problem Set 2
(due Wednesday, September 18)

Fall 2013 Professor Jensen

Consider a N-firm Cournot oligopoly producing an identical product with (market) inverse demand function f. If q_i is firm i's output, then $Q = \Sigma_i q_i$ is total production and market price P is given by $P = f(Q)$. Each firm has the same total cost function $C(q_i)$.

Assume the following:

(A1) Demand is twice differentiable, bounded below, $f'(Q) < 0$, $f(0) > 0$, and there is a $K > 0$ such that $f(Q) = 0$ for all $Q \geq K$.

(A2) Each firm's cost function is twice differentiable and monotonically increasing, $C'(q_i) > 0$ for all $q_i > 0$, and $C(0) = 0$.

(A3) Each firm's marginal profit is decreasing in its own output, $2f'(Q) + f''(Q)q_i - C''(q_i) < 0$.

1. Under what conditions does a symmetric Nash (Cournot) equilibrium in outputs, $q_i^* = q$ for all $i=1,\ldots,N$, exist? It may be that (A1)-(A3) are sufficient. If not, explain what other assumption(s) is (are) needed.

ENORMOUS HINT: You are looking for a solution to the system of equations below.

$$f(Q) + f'(Q)q_i - C'(q_i) = 0$$
$$Q = Nq$$

2. Under what conditions is this symmetric Nash (Cournot) equilibrium unique? It may be that (A1)-(A3) are sufficient. If not, explain what other assumption(s) is (are) needed. Is $q > 0$?

3. Under what conditions does this symmetric Nash (Cournot) equilibrium converge to the perfectly competitive outcome as $N \to \infty$? Again, it may be that (A1)-(A3) are sufficient. If not, explain what other assumption(s) is (are) needed.