

DUE: Wednesday, April 20

Consider the pollution/aid game in Chambers-Jensen with the following utility functions:

$$V^n = a(y^n - A) - (1/2)b(y^n - A)^2 - cE + (1/2)dE^2 + eA - (1/2)fA^2 - g(y^n - A)E,$$

where the contribution function is  $C(A) = eA - (1/2)fA^2$ , and

$$V^s = h(y^s + A) - (1/2)i(y^s + A)^2 + jE - (1/2)kE^2 - m(y^s + A)E,$$

where  $j$  is the parameter that distinguishes a tough Southern government from a weak one. Specifically, to capture the fact that it is more difficult for a weak South to enforce, we assume that  $j$  is either  $j_T$  or  $j_W$ , where  $j_T < j_W$ , so  $\partial V^s / \partial E = jE - kE$  is greater for a weak South.

Finally, all constants  $a, b, \dots, j, k, m$  are positive.

1. Determine the Nash equilibrium  $(A^*, E^*)$  in a static game with certainty where North chooses aid to maximize utility and South chooses emissions to maximize utility.

NOTE: You only need to do this once. The equilibrium values differ for the two cases (South tough or weak) only in that  $j = j_T$  when South is tough and  $j = j_W$  when South is weak.

2. How do the equilibrium values of aid and emissions vary with the parameters  $e$  and  $j$ ? Interpret these results.

3. How do the equilibrium levels of Northern utility and Southern utility vary with the parameters  $e$  and  $j$ ? Interpret these results.

4. Now determine the Bayesian Nash equilibrium  $(A^*(\alpha), E_T^*(\alpha), E_W^*(\alpha))$  in a static game where North chooses  $A$  to maximize utility and Southern types  $T$  and  $W$  choose  $E$  to maximize utility assuming it is common knowledge that South is weak with probability  $\alpha$ .

5. How do the equilibrium values of aid and emissions vary with the probability  $\alpha$ ? Interpret these results.

6. How do the equilibrium levels of Northern and Southern utility vary with the probability  $\alpha$ ? Interpret these results.