Success Probabilities in Gauss-Poisson Networks with and without Cooperation

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Abstract—**Gauss-Poisson processes (GPPs) are a class of clustered point processes, which include the Poisson point process as a special case and have a simpler structure than general Poisson cluster point processes. In this paper, we propose the GPP as a model for wireless networks that exhibit clustering behavior. We calculate the success probabilities and provide their bounds for three kinds of GPP networks: (1) the basic model where the desired transmitter is independent of the GPP and all nodes in the GPP are interferers; (2) the non-cooperative model where the desired transmitter is one of the nodes in the GPP; (3) the cooperative model where both nodes in a two-node cluster of the GPP serve a receiver cooperatively using non-coherent joint transmission. Our results show that the bounds, especially the upper bounds, provide useful approximations that well fit the actual success probability for different operating regimes.**

I. Introduction

A. Motivation

Stochastic geometry tools have been widely used to analyze wireless networks; see, e.g., [1]. Most existing works in the literature model the wireless networks using the Poisson point process (PPP) due to its tractability for analysis. But the PPP modeling methodology may be inadequate for certain scenarios where the spatial distribution of transceivers are less likely to be independent from each other.

In some circumstances, the transmitters form clusters, due to geographical factors (e.g. access points inside a building), or population factors (e.g. base stations in urban regions), or MAC protocols. Cluster point processes are thus suitable for modeling transmitter locations in those circumstances. A few prior studies have treated models of cluster point processes, for example the Neyman-Scott process [2] [3]. In those works dealing with cluster point process models, the system performance indicators, such as success probability and mean achievable rate, are usually in complex form involving multiple integrals.

In this paper, we focus on the Gauss-Poisson process (GPP), which is a relatively simple cluster point process with either one or two points in a cluster. As such, it retains a good level of tractability and constitutes a definitive improvement over the PPP in cases where "attraction" exists between node locations.

B. Related Work

The GPP has been primarily studied in mathematical statistics. In [4], stationarity, ergodicity, and infinite divisibility of the GPP have been characterized, and connections with other classes of point processes have been discussed. In [5], a simple method for simulating the GPP has been proposed, given its intensity and pair correlation function, and therein it has been also shown that the GPP can be used to generate stationary point processes with almost arbitrary two-point correlation function ξ (as long as it is clustered).[1] This makes the GPP a highly versatile model.

The cooperation technique considered in this paper belongs to joint processing and is called joint transmission (JT). Both coherent and non-coherent JT schemes exist. In [6], coherent JT has been studied for cellular networks in which base stations cooperate in a pairwise manner. In our cooperative model, we consider pairwise transmitter cooperation of non-coherent JT, where the transmitters are modeled as the GPP and the receiver uses soft-combining [7] (also called delay-diversity combining).

C. Contributions

We derive and bound the success probabilities for the following three network models: (a) The basic model: the desired transmitter is independent of the GPP. (b) The non-cooperative model: the desired transmitter is a point of the GPP, and the other point in the same cluster acts as an interferer. (c) The cooperative model: the desired transmitter is a point of the GPP, and the other point in the same cluster (if any) acts as a cooperator.

From a broader perspective, the contribution of the paper lies in the investigation of the benefits of cooperative communications in the context of a larger network, i.e., in the presence of interference from other nodes.

II. System Models

Definition 1 (Gauss-Poisson process [1, Example 3.8]). *A GPP is a Poisson cluster process with homogeneous independent clustering. The intensity of the parent process is denoted by λ_{p}. Each cluster has one or two points, with probabilities $1 - p$ and p, respectively. If a cluster has one point, it is located exactly at the location of the parent. If a cluster has two points, one of them is at the location of the parent, and the other is uniformly distributed within the circle with radius u centered at the location of the parent.*

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[1]The function ξ quantifies the two-point correlation in excess of the Poisson distributed points, and is related to the second moment density $\rho^{(2)}(r)$ [1, Def. 6.5] by $\xi(r) \triangleq \rho^{(2)}(r)/\lambda^2 - 1$.

We model the locations of the transmitters as a GPP on $\mathbb{R}^2$. Without loss of generality, we set $u = 1$, while the results can be readily extended to the case where u is a random variable. In asymptotic analysis, we also consider the regime of $u \to 0$. Denote the parent point process by $\Phi_\mathrm{p} = \{x_1, x_2, ...\}$. Let $\{\Phi_i, i \in \mathbb{N}\}$ be the daughter processes, denoted by

$$\Phi_i = \begin{cases} \{o\} & \text{w.p. } 1 - p \\ \{o, z_i\} & \text{w.p. } p \end{cases}, \quad (1)$$

where z_i is independently and uniformly distributed within the circle centered at the origin $o = (0,0)$ with radius u. The GPP is the union of the translated clusters:

$$\Phi = \bigcup_{i:x_i \in \Phi_\mathrm{p}} (\Phi_i + x_i).$$

Our analysis is focused on a typical receiver located at the origin o with desired transmitter at $x_0 = (b, 0)$ with $b \neq 0$.

We adopt a path loss model $\ell(x) = \|x\|^{-\alpha}$, where $x \in \mathbb{R}^2$ and $\alpha > 2$, and assume the power fading coefficients to be spatially independent with exponential distribution of mean unity (i.e., Rayleigh fading). Denote by h_x the power fading coefficient between the transmitter x and the receiver at o. We set all transmit powers to unity and focus on the interference-limited regime thus omitting the thermal noise.

A. Basic Model

The desired transmitter x_0 is independent of the GPP, and all points in the GPP are considered as interferers. In this case, the signal-to-interference-ratio (SIR) at the receiver located at the origin o is

$$\text{SIR} = \frac{h_0 b^{-\alpha}}{\sum_{x \in \Phi} h_x \|x\|^{-\alpha}}, \quad (2)$$

where h_0 is the power fading coefficient between the desired transmitter and the receiver.

B. Non-cooperative Model

In this case, the desired transmitter x_0 is taken as a parent point from the GPP. All points except x_0 in the GPP are interferers. Therefore, there is an interferer near the desired transmitter with probability p, and the SIR at the receiver is

$$\text{SIR} = \frac{h_{x_0} b^{-\alpha}}{\sum_{x \in \Phi \setminus \{x_0\}} h_x \|x\|^{-\alpha}}. \quad (3)$$

C. Cooperative Model

In this case, the desired transmitter x_0 is taken from a cluster Φ_0 of the GPP, and if there is another point in Φ_0, it acts as a cooperator. We assume that if there is a cooperator, the receiver uses soft-combining [7], which combines the signals from the two transmitters by accumulating the power. In this way, the receiver is served by both transmitters in a cluster, and all points from other clusters of the GPP act as interferers.

The SIR at the receiver is

$$\text{SIR} = \frac{\sum_{x \in \Phi_0} h_x \|x\|^{-\alpha}}{\sum_{x \in \Phi \setminus \Phi_0} h_x \|x\|^{-\alpha}}. \quad (4)$$

III. SUCCESS PROBABILITY

We assume that the receiver can decode its signal successfully if its SIR exceeds a threshold θ. The interference is denoted by I, which is the denominator of the SIR expression in (2), (3) and (4) for the three models, respectively. In this section, we derive the success probabilities of the three GPP network models.

A. Basic Model

Lemma 1. *Let* $v : \mathbb{R}^2 \mapsto [0, 1]$ *be a measurable function such that* $1 - v$ *has bounded support. Then the probability generating functional (PGFL) of the GPP is*

$$G[v] = \exp\left(\lambda_\mathrm{p} \int_{\mathbb{R}^2} \left[(1 - p)v(x)\right.\right.$$
$$\left.\left. + pv(x)\frac{1}{2\pi}\int_0^{2\pi} v(x + w(\psi))d\psi - 1\right]dx\right), \quad (5)$$

where $w(\psi) = (\cos\psi, \sin\psi)$.

Proof: The PGFL of Poisson cluster processes is (see, e.g., [1, Cor. 4.12])

$$G^{\mathrm{PCP}}[v] = \exp\left(\lambda_\mathrm{p} \int_{\mathbb{R}^2} (G_0^{[x]}[v] - 1)dx\right), \quad (6)$$

where $G_0^{[x]}[v]$ is the PGFL of the cluster $\Phi^{[x]}$ that is centered at x, given by $G_0^{[x]}[v] = \mathbb{E}\left(\prod_{y \in \Phi^{[x]}} v(y)\right)$.

In the GPP, there are either one or two points in a cluster. According to its definition, we have

$$G_0^{[x]}[v] = (1 - p)v(x) + p\int_0^{2\pi} v(x)v(x + w(\psi))\frac{1}{2\pi}d\psi. \quad (7)$$

Substituting (7) into (6), we obtain (5). ∎

Theorem 1. *In the basic model, the success probability is the Laplace transform of the interference I at θb^α, i.e.,*

$$P_\mathrm{s}(\theta, b, \lambda_\mathrm{p}, \alpha, p) = \mathcal{L}_I(\theta b^\alpha), \quad (8)$$
$$\mathcal{L}_I(s) = \exp\left(2\pi\lambda_\mathrm{p}\int_0^\infty \left(\frac{1 - p}{1 + sr^{-\alpha}} + \frac{p}{1 + sr^{-\alpha}}\frac{1}{2\pi}\right.\right.$$
$$\left.\left. \cdot \int_0^{2\pi} \frac{1}{1 + s(r^2 + 1 + 2r\cos\psi)^{-\alpha/2}}d\psi - 1\right)rdr\right). \quad (9)$$

Proof: The success probability P_s is a function of $\theta, b, \lambda_\mathrm{p}, \alpha$ and p, and

$$P_\mathrm{s}(\theta, b, \lambda_\mathrm{p}, \alpha, p) = \mathbb{P}\left(\frac{h_0 b^{-\alpha}}{I} > \theta\right) \overset{(a)}{=} \mathcal{L}_I(\theta b^\alpha), \quad (10)$$

where (a) follows because $h_0 \sim \exp(1)$. The Laplace transform of I is derived by the PGFL as follows:

$$\mathcal{L}_I(s) = \mathbb{E}_{\Phi, \{h\}}\left(\exp\left(-\sum_{x \in \Phi} sh_x\|x\|^{-\alpha}\right)\right)$$
$$= \mathbb{E}_\Phi\left(\prod_{x \in \Phi} \mathbb{E}_h\left(\exp(-sh\|x\|^{-\alpha})\right)\right)$$
$$= \exp\left(\lambda_\mathrm{p}\int_{\mathbb{R}^2}\left[(1 - p)v(x)\right.\right.$$
$$\left.\left. + pv(x)\int_0^{2\pi} v(x + w(\psi))\frac{1}{2\pi}d\psi - 1\right]dx\right), \quad (11)$$

where

$$v(x) = \mathbb{E}_h\big(\exp(-sh\|x\|^{-\alpha})\big) = \frac{1}{1 + s\|x\|^{-\alpha}}. \qquad (12)$$

Substitute (12) into (11), we obtain (9). ∎

Though the success probability is not in closed form, it has bounds in closed form for $\alpha = 4$.

Corollary 1. *For $\alpha = 4$, $P_s(\theta, b, \lambda_p, 4, p)$ has upper and lower bounds in closed form, as follows:*

$$P_s(\theta, b, \lambda_p, 4, p) \leq \exp\left(-\frac{\pi^2}{2}\lambda_p(1-p)\sqrt{s} + \lambda_p p W_u(s)\right),$$

and

$$P_s(\theta, b, \lambda_p, 4, p) \geq \exp\left(-\frac{\pi^2}{2}\lambda_p(1-p)\sqrt{s} + \lambda_p p W_l(s)\right),$$

where $s = \theta b^4$,

$$W_u(s) = \frac{\pi\sqrt{s}}{4(9s^2 + 40s + 16)}\left(8s^{\frac{3}{2}}\ln\frac{s}{s+4} + (-3s^2 + 24s + 16)\arctan\frac{2}{\sqrt{s}} - \pi(\frac{21}{2}s^2 + 48s + 24)\right) + \frac{\pi\sqrt{s}}{2(4s+1)}$$
$$\cdot\left(s^{\frac{3}{2}}\ln\frac{s}{s+1} + (3s+1)(\arctan\frac{1}{\sqrt{s}} - \pi)\right), \qquad (13)$$

and

$$W_l(s) = -\frac{\pi^2\sqrt{s}}{4} - \frac{\pi s^{\frac{1}{4}}}{8}\left(2\sqrt{2}\pi - \sqrt{2}\ln\frac{1 + \sqrt{s} - \sqrt{2}s^{\frac{1}{4}}}{1 + \sqrt{s} + \sqrt{2}s^{\frac{1}{4}}}\right.$$
$$\left. - 2(\sqrt{2} + 2s^{\frac{1}{4}})\arctan\frac{-\sqrt{2} - s^{\frac{1}{4}}}{s^{\frac{1}{4}}} - 2(\sqrt{2} - 2s^{\frac{1}{4}})\right.$$
$$\left. \cdot\arctan\frac{-\sqrt{2} + s^{\frac{1}{4}}}{s^{\frac{1}{4}}}\right) + \frac{2\pi s^{\frac{3}{2}} + \pi\sqrt{s}}{2(4s+1)}\arctan\frac{1}{\sqrt{s}}$$
$$+ \frac{\pi s^2}{4s+1}\ln\frac{s}{s+1} - \frac{2\pi^2 s^{\frac{3}{2}} + \pi^2\sqrt{s}}{2(4s+1)}. \qquad (14)$$

Proof: Omitted due to space constraints. ∎

The following corollary gives the success probability in the asymptotic regime $u \to 0$.

Corollary 2. *In the basic model, the success probability in the asymptotic regime $u \to 0$ is equal to the Laplace transform of the interference I_0 at θb^α, i.e.,*

$$P_s(\theta, b, \lambda_p, \alpha, p) = \mathcal{L}_{I_0}(s), \qquad (15)$$

where $s = \theta b^\alpha$ and

$$\mathcal{L}_{I_0}(s) = \exp\left(-\frac{2\pi^2\lambda_p s^{\frac{2}{\alpha}}}{\alpha\sin\left(\frac{2\pi}{\alpha}\right)}\left(1 + \frac{2p}{\alpha}\right)\right). \qquad (16)$$

Proof: Let $\Phi_1 \in \Phi_p$ be the set of parent points of the clusters with only one point in the GPP and $\Phi_2 = \Phi_p\backslash\Phi_1$ be the set of parent points of the clusters with co-located two points in the GPP. Therefore, Φ_1 and Φ_2 are two independent PPPs with intensities $(1-p)\lambda_p$ and $p\lambda_p$ respectively. Let $I_1 = \sum_{x\in\Phi_1} h_x\|x\|^{-\alpha}$ and $I_2 = \sum_{x\in\Phi_2}(h_{x,1} + h_{x,2})\|x\|^{-\alpha}$ be the interference from Φ_1 and Φ_2 respectively, where $h_{x,1}$ and $h_{x,2}$ are the power fading coefficients between the two transmitters

co-located at x and the typical receiver. The Laplace transform of the interference is then given by

$$\mathcal{L}_{I_0}(s) = \mathbb{E}\big(\exp\big(-sI_1 - sI_2\big)\big),$$

which evaluates to (16). From (10), we get the success probability for the GPP in the limit of $u \to 0$. ∎

B. Non-cooperative Model

Lemma 2. *Conditioned on a point of the parent process being located at y, the conditional PGFL of the GPP excluding y is*

$$G_y[v] = G[v]\left(1 - p + p\frac{1}{2\pi}\int_0^{2\pi} v(y + w(\psi))d\psi\right), \qquad (17)$$

where $w(\psi) = (\cos\psi, \sin\psi)$.

Proof: Denote the points in the cluster whose center is the desired transmitter y as Φ_0, and all points in other clusters as $\Phi_c = \Phi\backslash\Phi_0$. From Slivnyak's theorem [1], conditioning on Φ_0 does not change the distribution of other clusters, and the distribution of the points excluding Φ_0 remains the same as the original GPP Φ. Thus, the conditional PGFL excluding y is

$$G_y[v] = \mathbb{E}\left(\prod_{x\in(\Phi_c\bigcup\Phi_0)\backslash\{y\}} v(x)\right)$$
$$= \mathbb{E}\left(\prod_{x\in\Phi_c} v(x)\right)\mathbb{E}\left(\prod_{x\in\Phi_0\backslash\{y\}} v(x)\right)$$
$$= \mathbb{E}\left(\prod_{x\in\Phi} v(x)\right)\mathbb{E}\left(\prod_{x\in\Phi_0\backslash\{y\}} v(x)\right)$$
$$= G[v]\left(1 - p + p\int_0^{2\pi} v(y + w(\psi))\frac{1}{2\pi}d\psi\right). \qquad (18)$$
∎

Theorem 2. *In the non-cooperative model, the success probability is the Laplace transform of the interference I at θb^α*

$$P_s(\theta, b, \lambda_p, \alpha, p) = \bar{\mathcal{L}}_I(s), \qquad (19)$$

where $s = \theta b^\alpha$ and

$$\bar{\mathcal{L}}_I(s) = \mathcal{L}_I(s) \cdot \left(1 - p + \frac{p}{2\pi}\int_0^{2\pi}\frac{1}{1 + s(b^2 + 1 + 2b\cos\psi)^{-\alpha/2}}d\psi\right). \qquad (20)$$

Proof: The proof is similar to that of Theorem 1, with the conditional PGFL $G_y[v]$ instead of $G[v]$ used. ∎

Similar to the basic model, $P_s(\theta, b, \lambda_p, \alpha, p)$ is not in closed form. For $\alpha = 4$, however, closed-form lower and upper bounds are available.

Corollary 3. *For $\alpha = 4$, $P_s(\theta, b, \lambda_p, 4, p)$ has upper and lower bounds in closed form, as follows:*

$$P_s(\theta, b, \lambda_p, 4, p) \leq \exp\left(-\frac{\pi^2}{2}\lambda_p(1-p)\sqrt{s} + \lambda_p p W_u(s)\right)$$
$$\cdot\left(1 - p + \frac{p}{2}\left(\frac{1}{1 + s(b^2 + 1 + 2b)^{-2}} + \frac{1}{1 + s(b^2 + 1)^{-2}}\right)\right),$$

and

$$P_s(\theta, b, \lambda_p, 4, p) \geq \exp\left(-\frac{\pi^2}{2}\lambda_p(1-p)\sqrt{s} + \lambda_p p W_l(s)\right)$$
$$\cdot \left(1 - p + \frac{p}{2}\left(\frac{1}{1 + s(b^2 + 1 - 2b)^{-2}} + \frac{1}{1 + s(b^2 + 1)^{-2}}\right)\right),$$

where $s = \theta b^4$.

Proof: The proof is based on Corollary 1 and utilizes the property of $0 \leq \cos\psi \leq 1$ for $\psi \in [-\pi/2, \pi/2]$ and $-1 \leq \cos\psi \leq 0$ for $\psi \in [\pi/2, 3\pi/2]$. ∎

The success probability in the asymptotic regime $u \to 0$ is given by the following corollary.

Corollary 4. *In the non-cooperative model, the success probability as $u \to 0$ is equal to the Laplace transform of the interference I_0 at θb^α, i.e.,*

$$P_s(\theta, b, \lambda_p, \alpha, p) = \bar{\mathcal{L}}_{I_0}(s), \qquad (21)$$

where $s = \theta b^\alpha$ *and*

$$\bar{\mathcal{L}}_{I_0}(s) = \frac{b^\alpha + (1-p)s}{b^\alpha + s}\exp\left(-\frac{2\pi^2\lambda_p s^{\frac{2}{\alpha}}}{\alpha\sin\left(\frac{2\pi}{\alpha}\right)}\left(1 + \frac{2p}{\alpha}\right)\right).$$

Proof: The proof follows the same line as that of Corollary 2 except for the conditional PGFL of the GPP as $u \to 0$. ∎

C. Cooperative Model

In the cooperative model, the cooperator transmits the same infomation as the desired transmitter simultaneously. In this case, the received power, denoted by P_w, is the sum of the received signal power from the desired transmitter and the cooperator, i.e.,

$$P_w = \begin{cases} hb^{-\alpha} & \text{w.p. } 1-p, \\ h_1 b^{-\alpha} + h_2 c^{-\alpha} & \text{w.p. } p, \end{cases} \qquad (22)$$

where $h, h_1, h_2 \sim \exp(1)$ are mutually independent, and $c = \sqrt{b^2 + 1 + 2b\cos\psi}$, $\psi \sim \text{unif}(0, 2\pi)$.

The exponential distribution has the property that if $h \sim \exp(1)$, then $lh \sim \exp(1/l)$, for $l > 0$. Thus, conditioned on c, for the case where $P_w = h_1 b^{-\alpha}$, $P_w \sim \exp(b^\alpha)$; for the case where $P_w = h_1 b^{-\alpha} + h_2 c^{-\alpha}$, if $b \neq c$, P_w follows the hypoexponential distribution $\text{Hypo}(b^\alpha, c^\alpha)$ and the PDF of P_w is $f_P(x) = \frac{b^\alpha c^\alpha}{b^\alpha - c^\alpha}\left(\exp(-c^\alpha x) - \exp(-b^\alpha x)\right)$, otherwise, $P_w \sim \text{Erlang}(2, b^\alpha)$ and $f_P(x) = b^{2\alpha} x \exp(-b^\alpha x)$.

Conditioned on that the center of a cluster is located at y, the conditional PGFL of the GPP, excluding that cluster is

$$\tilde{G}_y[v] = G[v]. \qquad (23)$$

This can be readily proved using Slivnyak's theorem.

Theorem 3. *In the cooperative model, the success probability is*

$$P_s(\theta, b, \lambda_p, \alpha, p) = (1-p)\mathcal{L}_I(\theta b^\alpha) + p\mathbb{E}_c(H(c)) \qquad (24)$$

where $H(c) = \frac{b^\alpha}{b^\alpha - c^\alpha}\mathcal{L}_I(\theta c^\alpha) - \frac{c^\alpha}{b^\alpha - c^\alpha}\mathcal{L}_I(\theta b^\alpha)$ *and the PDF of c is* $f_c(x) = \frac{2x}{\pi\sqrt{(x^2 - (b-1)^2)((b+1)^2 - x^2)}}$ *over the interval* $[|b-1|, b+1]$.

Proof: The SIR at the receiver is P_w/I. We have

$$P_s(\theta, b, \lambda_p, \alpha, p) = \mathbb{E}_{P_w}\mathbb{P}\left(\frac{P_w}{I} > \theta\right)$$
$$= (1-p)\mathbb{P}\left(\frac{hb^{-\alpha}}{I} > \theta\right) + p\mathbb{P}\left(\frac{h_1 b^{-\alpha} + h_2 c^{-\alpha}}{I} > \theta\right)$$
$$= (1-p)\mathcal{L}_I(\theta b^\alpha) + pQ, \qquad (25)$$

where $Q \triangleq \mathbb{P}(\frac{h_1 b^{-\alpha} + h_2 c^{-\alpha}}{I} > \theta)$. Since the case of $c = b$ has a vanishing probability thus contributing zero to Q, we have $Q = \mathbb{E}_{I,c}\left(\frac{b^\alpha}{b^\alpha - c^\alpha}\exp(-\theta c^\alpha I) - \frac{c^\alpha}{b^\alpha - c^\alpha}\exp(-\theta b^\alpha I)\right) = \mathbb{E}_c(H(c))$.

As $c = \sqrt{b^2 + 1 + 2b\cos\psi}$, where $\psi \sim \text{unif}(0, 2\pi)$, the CDF and the PDF of c can be obtained directly. ∎

For $\alpha = 4$, upper and lower bounds of the success probability can be derived.

Corollary 5. *For $\alpha = 4$ and $b \neq 1/2$, $P_s(\theta, b, \lambda_p, 4, p)$ has upper and lower bounds, as follows:*

$$P_s(\theta, b, \lambda_p, 4, p) \leq \left(1 - p - \frac{p|b-1|^4}{b^4 - |b-1|^4}\right)\mathcal{L}_I(\theta b^4)$$
$$+ \frac{pb^4}{b^4 - |b-1|^4}\mathcal{L}_I(\theta|b-1|^4), \qquad (26)$$

and

$$P_s(\theta, b, \lambda_p, 4, p) \geq \left(1 - p - \frac{p(b+1)^4}{b^4 - (b+1)^4}\right)\mathcal{L}_I(\theta b^4)$$
$$+ \frac{pb^4}{b^4 - (b+1)^4}\mathcal{L}_I(\theta(b+1)^4). \qquad (27)$$

Proof: The proof is based on Theorem 3 and the property of $|b-1| \leq c \leq b+1$. ∎

To get the bounds in closed form, we may apply (8) and Corollary 1 to (26) and (27). In (26), we use the upper bound of $\mathcal{L}_I(\cdot)$ if the coefficient of $\mathcal{L}_I(\cdot)$ is larger than 0, and use the lower bound of $\mathcal{L}_I(\cdot)$ otherwise. While in (27), we use the lower bound of $\mathcal{L}_I(\cdot)$ if the coefficient of $\mathcal{L}_I(\cdot)$ is larger than 0, and use the upper bound of $\mathcal{L}_I(\cdot)$ otherwise.

It is worth noting that if $p = 0$, the GPP reduces to the PPP with intensity $\lambda = \lambda_p$. Substituting $p = 0$ into Theorems 1-3, we obtain the same result

$$P_s(\theta) = \exp\left(-\pi\lambda_p\theta^\delta b^2\Gamma(1+\delta)\Gamma(1-\delta)\right), \qquad (28)$$

where $\delta \triangleq 2/\alpha$. The result coincides with that in [1, Ch. 5.2].

IV. NUMERICAL RESULTS

The numerical results are obtained according to the analytical results we have derived.

Figure 1 shows the success probability and closed-form bounds of the basic model as a function of the distance between the receiver and the desired transmitter. We observe that the success probability decreases with increasing distance b. We also observe that the upper bounds are satisfactorily tight for the basic model.

Figure 2 shows the success probability and closed-form bounds of the non-cooperative model as a function of the distance between the receiver and the desired transmitter. We
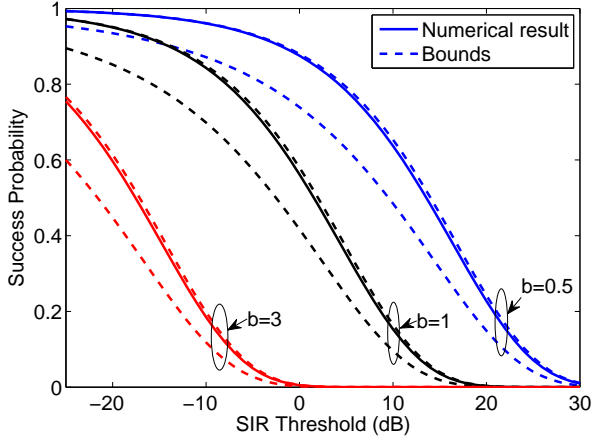
Fig. 1. The success probability and closed-form bounds with varying distance between the receiver and the desired transmitter for the basic model ($\lambda_{\mathrm{P}} = 0.1, p = 0.5, \alpha = 4$).
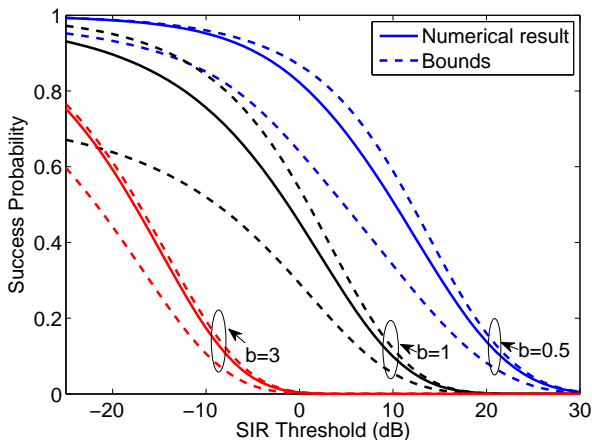

Fig. 3. The success probability and closed-form bounds with varying distance between the receiver and the desired transmitter for the cooperative model ($\lambda_{\mathrm{P}} = 0.1, p = 0.5, \alpha = 4$).

observe that the upper bounds are tight for both large and small values of b. However, when b approaches 1, which is the distance between two points in a two-point cluster, the bounds become loose.

Fig. 2. The success probability and closed-form bounds with varying distance between the receiver and the desired transmitter for the non-cooperative model ($\lambda_{\mathrm{P}} = 0.1, p = 0.5, \alpha = 4$).

Figure 3 shows the success probability and closed-form bounds of the cooperative model as a function of the distance between the receiver and the desired transmitter. We also observe that the bounds are tight for both large and small values of b, while they become loose when b is close to 1.

Figure 4 compares the success probability curves of the three models. We observe that the performance of the non-cooperative model is the worst while that of the cooperative model is the best, with a gap of $4 \sim 5$ dB.

V. CONCLUSION

In this paper, we proposed the application of the GPP in several different wireless network models, and derived the success probabilities and their bounds for the considered
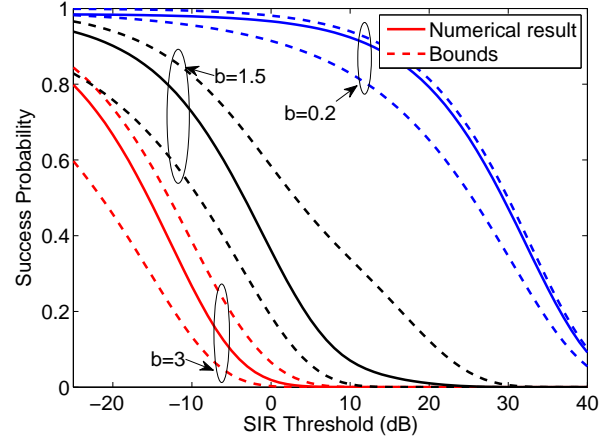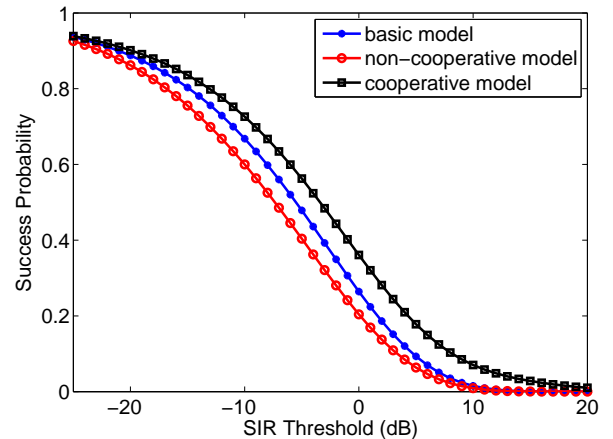

Fig. 4. The success probabilities of the basic model, the non-cooperative model, and the cooperative model ($\lambda_{\mathrm{P}} = 0.1, p = 0.5, b = 1.5, \alpha = 4$).

models. The results indicate that the bounds, especially the upper bounds, provide useful approximations that well fit the actual success probability for different operating regimes.

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