Two of our greatest assets as mathematicians are that we are experts at problem solving and we have insight into how mathematics is learned. These are skills that would be of great benefit to our students if we could transmit our knowledge of them in the classroom. Although our interest in mathematics is different from that of our students, they are already involved in learning mathematics and they will undoubtedly have problems to solve throughout the course of their lifetime.

There is however a gap to be bridged between us and our students. On the one hand, although we have these skills, we are often not aware of the thought processes that are behind them; we simply solve the problems and learn the mathematics. On the other hand, our students have often never contemplated using problem solving techniques or learning strategies that we employ automatically. Furthermore, our students do not see mathematics as we do. They often do not share our motivation and are seeing the material on hand for the first time. To bridge this gap, I suggest trying to develop an awareness of how we learn and how we solve problems, keeping in mind the student perspective.

Having developed some insight into the general principles of learning and problem solving, we can build expositions of problem solving techniques into examples in class and use our knowledge of the learning process to make our classroom more conducive to learning.

The first topic we explored was our motivation to study math. Many reasons surfaced, among them: because we are successful at it, because it is a language for abstract thoughts, we like the aha moment, we like problem solving. On exploring the student point of view, we noted that a student who didn't experience any of these positive rewards of studying mathematics might see us as a bunch of zealots and will not share our motivation. So where possible, if we can create an opportunity for our students to share these motivational experiences, we should. Although most of our students will not become mathematicians, it is important to provide all with the motivation to learn.

In order for a mathematician to explore the general principles of learning, with the student perspective in mind, it might help to change the context, since we are already experts at mathematics. The question of what principles of learning are transferable arose and it was suggested that perhaps it is possible to find some general principles of learning that are applicable to many disciplines. Indeed since our students are often quite competent in other fields, we can help them apply their learning skills to mathematics, if we can bring the relevant skills to light for them.

In order to explore the learning process in a different context, we tried to learn to juggle. There was already a range of capabilities among the students. It was agreed that a demonstration was not enough to learn the skill; a student would have to actually try for themselves. It was then generally agreed that some directions were necessary and steps were provided. Some students were afraid to try for fear of failing to be able to juggle in front of the class and some who were eager to learn hesitated to leave their seats to give themselves the necessary space to practice. After a little encouragement most people followed the steps and completed a few of the steps involved in learning. Those who already knew how to juggle helped those who were trying to learn. It was generally agreed that more practice was necessary, but some people felt that they might indeed be capable of juggling in the near future.

We then reflected on the learning process just experienced. One of the questions discussed was, "How can we create a classroom atmosphere, in which people will feel relaxed enough to learn and help each other". It was generally agreed that students needed to be able to relax in the classroom before they could learn. The instructor might facilitate this might be by trying to be open and relaxed themselves. This led to the question of how to make students who were afraid of making a fool of themselves in front of the class or the instructor feel at ease. Suggestions on how to deal with questions from students were given. It was suggested that the instructor should answer all questions in the same manner, all questions should be greeted with interest, it should be acknowledged that mistakes were an

acceptable part of the learning process and if a student makes a mistake which is common, it provides an opportunity to warn people that this is a common pitfall.

To conclude, mathematicians were urged to explore the learning process in a variety of contexts. They were encouraged to sharpen their awareness of the both the problem solving process and the learning process in order to gain some insight into the student perspective and to use their insight to benefit their students.

Below I have included a number of questions that one might ponder on as they contemplate their experiences.

The Learning Process

As you learn, here are some things to pay attention to.....that is if you can find a way to pay attention to what you are thinking and think at the same time.....try zen :)

1. What happens to your attention? How difficult is it to be aware of what happens to your attention? Are we in control of where our attention goes? What draws your attention?

2. Do you look for patterns when learning? Do you have to look for patterns? Does your mind have a tendency to pick out patterns or impose patterns?

3. How important is repetition in learning and in reinforcing what you have learned?

4. Does it help to have a checklist of goals so you can mark off your achievements? How important is feedback?

5. What rewards do you get from learning that motivate you to learn more?

6. What is it in a teachers behavior that helps you, or repels you? How about their sense of humor, their mastery of the subject at hand, their openness to questions, their encouragement?

7. Do you reflect behaviors? When a teacher is relaxed, do you feel relaxed? When a teacher is tense, do you feel tense?

8. How important is relaxation to the learning process? Are there different states of relaxation ? How do you achieve that optimal state of relaxation and alertness to maximize your learning capacity? Check out the concept of *Flow* by Mihly Cskszentmihlyi.

9. How important is personalization of the material? Can you pick up the concept by being a spectator or is it something that requires exploration, practice? Does it help to explore how the concept relates to your own environment?

10. Does it help to connect the concept to things you have learned before? How often does this happen in learning? Is there a lot of interaction between disciplines?

11. Can you transfer your learning skills from one subject to another? How about writing out notes after your music lesson or keeping a log book on your mental weight lifting?

The Problem Solving Process

How important is it to clarify exactly what the problem is before you try to find a solution? What goes into the process of formulating the problem? Do you write it down? Does it take many attempts? Do you personalize it? Do you think of concrete examples?.....

Finding a Solution

1. How important is it to have a well formulated problem? Do you review the statement of the problem periodically during the process of problem solving?

2. Do you give yourself lots of space to work?

3. Do you write down known results that the problem reminds you of? How does it help to have these written down?

4. Do you devise a plan of action for the global picture, or do you work on local aspects or both? Do you reformulate your plan every now and then? How important is it to formulate a plan?

5. Do you draw pictures?

6. How important is it to just relax and play with the problem? Do solutions come to you more frequently in a relaxed state of mind? Can you recreate the optimal solution producing state of mind at will?

7. Can you solve problems in a classroom? under pressure?

8. Is that AHA moment totally mysterious? Can you pick out a pattern in what precedes it when it happens?

9. Do you work lots of examples? Do examples help?

10. Do you draw analogies with other problems?

11. What motivates you, is it the calculations, seeing the big picture, the aha moment, the million dollar prize, the attention ....finishing graduate school?

12. Do you have to be brave to solve the problem? How big is the step from solving something that you've seen an example of to solving something new (and completely different)?

Books Worth Reading

Csikszentmihalyi, Mihaly, Flow.

Lakatos, Imre, Proofs and Refutations

Mayer, Richard Thinking, Problem Solving and Cognition.

Polya, George Mathematical discovery; on understanding, learning, and teaching problem solving.

Polya, George Mathematics and plausible reasoning.

Polya, George How to solve it : a new aspect of mathematical method