Math 10260 Projects – Fall 2008

The **Goal** of these projects is to give you the opportunity to make your own connections between mathematics and modern society by considering a wide variety of problems ranging from economic and environmental issues to social and political situations that can be modeled and solved by mathematical means. They will help you establish connections between Math 10260 and your other courses, and they will allow you to make contributions in areas of your interest and expertise. In addition, they will provide you with an opportunity to interact and collaborate with your classmates.

The **Topic** can be chosen from:

a. **Chapters 5–11 Projects in the book**.
b. The projects described below under "Other Project Topics".
c. Other courses you are able to establish a connection with math 10260.
d. Anything that you find interesting and is approved by your teacher.

The **Rules** are:

I. You can work in groups of size 1-3 students (from any section of Math 10260).
II. Each group submits one (typed) paper (and e-copy if possible).
III. Each member of the group receives the same score–a number between 0 and 10, in which will count toward your 20 participation points.
IV. The first draft is due by **Nov 10** and final version due by **Dec 08**. You must state in your project stating your **project title**, the **names** of your team members and the **class sections** each member is from.

Other Project Topics

(1) **The Social Security.** Some experts project that the Social Security shortfall over the next 75 years will be about four trillion dollars. Is that true? How do they know? Make your contribution in the national debate about saving Social Security using ideas and techniques you learned in Math 10260 (for example, income streams).

(2) **The Deficit.** Visit the Webpage of the Congressional Budget Office (CBO) at http://www.cbo.gov/ and try to make sense of the numbers you will find in "Current Budget Projections". Note that income streams are useful in making projections.

(3) **Arctic National Wildlife Refuge: To drill or not to drill?** A question for public debate these days is whether the Arctic National Wildlife Refuge (ANWR) contains enough oil to make its extraction worth both the economic cost and the environmental risk. Make your contribution by doing the numbers.

(4) **The Paradox of Choice.** In this book, Barry Schwartz, among many other things, claims that freedom of choice can turn into a tyranny of choice. He even uses some math to make his point. For example, in pages 67–73 he uses familiar curves to give a general explanation of how we go about evaluating options and making decisions. Write a report on this very interesting book and try to relate it to ideas you learned in Math 10260.

(5) Read carefully section 6.1 on consumer and producer surplus, compare it with writings in economic's literature, and explain how are demand and supply curves determined.

(6) **Flatland.** Imagine that you live in a plane (a 2D-space) and that you are not able to see 3D shapes. Then, think of ways for visualizing such shapes. A good source of ideas is the book "Flatland" by Edwin Abbott. Read this book and extend its ideas to describe how inhabitants of 3D-space (i.e., humans) could visualize 4D shapes.

(7) **A. Income distribution and Lorentz curves.** The way that income is distributed throughout a given society is an important object of study for economists. The U.S. Census Bureau collects and analyzes income data, which it makes available at its

website, www.census.gov. In 2001, for instance, the poorest 20% of the U.S. population received 3.5% of the money income, while the richest 20% received 50.1%. The *cumulative* proportions of population and income are shown in the following table:

proportion of population	**proportion of income**
0	0
0.20	0.035
0.40	0.123
0.60	0.268
0.80	0.499
1.00	1.00

For instance, the table shows that the lowest 40% of the population received 12.3% of the total income. We can think of the data in this table as being given by a functional equation $y = f(x)$, where x is the cumulative proportion of the population and y is the cumulative proportion of income. For instance, $f(0.60) = 0.268$ and $f(0.80) = 0.499$. Such a function (or, more properly speaking, its graph) is called a **Lorentz curve**.

(i) Show that $f(x) = 0.1x + 0.9x^2$ is a possible Lorentz curve. Also, compute the income received by the lowest 0 %, 50%, and 100% of the population.

(ii) Show that $f(x) = 0.3x + 0.9x^2$ is **not** a Lorentz curve.

(iii) For the Lorentz curve in (i) show the following properties:
(a) $f(0) = 0, f(1) = 1,$ and $0 \leq f(x) \leq 1$ for all $0 \leq x \leq 1$,
(b) $f(x)$ is an increasing function,
(c) $f(x) \leq x$ for all x, $0 \leq x \leq 1$.

(iv) Explain why properties (a)–(c) hold for every Lorentz curve.

(v) Write many other different formulas for Lorentz curves.

(vi) Using real data produce Lorentz curves for the U.S. in 2006.

(vii) Sketch the graph of a Lorentz curve and compare it with the line $y = x$.

B. Coefficient of Inequality. If the Lorentz curve of a country is given by $f(x) = x$ then its total income is distributed equally. Otherwise there are inequalities present in the distribution of income, which are measured by the following number:

$$\textbf{coefficient of inequality} = 2\int_0^1 [x - f(x)]dx,$$

which is also called the **Gini Index**.

(i) Compute the coefficient of inequality when $f(x) = 0.1x + 0.9x^2$.

(ii) Show that the Gini Index is the **ratio** of the area of the region between $y = f(x)$ and $y = x$ to the area of the region under $y = x$, and provide an economic inerpretation of this ratio.

(iii) Using real data estimate the Gini Index of the U.S. in 2006.

(8) **A.** (**The Cobb-Douglas Production Function**) Show that the production function $Q(K, L)$ having the properties:
(i) (Marginal Product of Capital) $\cdot$ (Capital) = $\alpha\cdot$ (Output),
(ii) (Marginal Product of Labor) $\cdot$ (Labor) = $(1 - \alpha)\cdot$ (Output),
for some constant α, $0 < \alpha < 1$, **must be** of the form $Q(K, L) = AK^{\alpha}L^{1-\alpha}$, for some constant A.

B. Read and understand the Solow Growth Model (Section 9.3) and do exercise 1, or 2, or 3 on page 607.

(9) **A.** You are 35 years old and your company offers you the following three retirement plans:

(Plan 1) At the beginning it deposits \$50,000 into an account A and nothing more during the next 30 years.

(Plan 2) For the next 30 years it deposits money continuously into an account B at a rate of $10,000$ dollars per year.

(Plan 3) At the age of 65 you will receive \$1,200,000 and nothing more during the next 30 years you will be working there.

If the accounts A and B yields 8% interest, compounded continuously, which option will you choose? Explain your answer.

B. Do part A again with interest rate at 10% compounded monthly. For Plan 2, assume that money will be deposited monthly into account B. To complete this part, you will have to set up a geometric series that gives the value of your retirement account. Go to your notes for continuous compounding and modify the set up for discrete compounding. Explain what each of the terms in the geometric series means. You should state clearly the first term, common ratio, and the formula you use to obtain the value of your retirement account from the geometric series. How would this change your decision in part A?

(10) **A.** A homeowner takes out a 20-year mortgage with an interest rate of 5% compounded continuously. The homeowner plans to make payments totalling \$1,500 per month. Let $M(t)$ be the amount owed after t years. Write an initial value problem modelling this situation. Then find the maximum amount of mortgage that the homeowner can afford.

B. Do part A again with interest rate at 5% compounded **quarterly**. To complete this part, you will have to set up a geometric series that gives the value of the mortgage. Go to your notes for continuous compounding and modify the set up for discrete compounding. Explain what each of the terms in the geometric series means. You should state clearly the first term, common ratio, and the formula you use to obtain the mortgage value from the geometric series. (Hint: You should prorate the interest because you are paying monthly.)

(11) Read carefully section 6.4 on population models and then do exercises 27 and 28 on page 445.

(12) **What does calculus have to do with change?** The two central concepts in calculus are the derivative (instantaneous rate of change) and the integral (total change). Write in your own words the way you understand these concepts. Give examples from mathematics and its applications to demonstrate them.