Vector bundles of finite order arose in the work of Griffiths and have been little studied since. They may provide the solution to the so-called "Oka principle with growth conditions" on affine complex manifolds. In this talk I will propose a new definition for such bundles using Finsler instead of Hermitian metrics. Then I will prove a vanishing theorem for the associated sheaf of finite order sections. This is the first small step from a larger program which will be outlined and which may lead to the proof of the Oka principle mentioned above.