We show a general convergence result for the spectrum of families of manifolds converging in the Lipschitz topology for metric pairs. As one of the applications, for a complete geometrically finite manifold M of bounded curvature we give both a lower bound for the bottom of the spectrum and an upper bound for the number of small eigenvalues. These bounds only depend on the dimension, the volume of the one-neighborhood of the convex core and curvature bounds.