

**Math 125, Final Exam**  
**December 16, 2004**

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
- **When told to begin, remove this answer sheet and keep it under the rest of your test. When told to stop, hand in just this one page.**
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

- |                         |                         |
|-------------------------|-------------------------|
| 1. (a) (b) (c) (d) (e)  | 15. (a) (b) (c) (d) (e) |
| 2. (a) (b) (c) (d) (e)  | 16. (a) (b) (c) (d) (e) |
| .....                   | .....                   |
| 3. (a) (b) (c) (d) (e)  | 17. (a) (b) (c) (d) (e) |
| 4. (a) (b) (c) (d) (e)  | 18. (a) (b) (c) (d) (e) |
| .....                   | .....                   |
| 5. (a) (b) (c) (d) (e)  | 19. (a) (b) (c) (d) (e) |
| 6. (a) (b) (c) (d) (e)  | 20. (a) (b) (c) (d) (e) |
| .....                   | .....                   |
| 7. (a) (b) (c) (d) (e)  | 21. (a) (b) (c) (d) (e) |
| 8. (a) (b) (c) (d) (e)  | 22. (a) (b) (c) (d) (e) |
| .....                   | .....                   |
| 9. (a) (b) (c) (d) (e)  | 23. (a) (b) (c) (d) (e) |
| 10. (a) (b) (c) (d) (e) | 24. (a) (b) (c) (d) (e) |
| .....                   | .....                   |
| 11. (a) (b) (c) (d) (e) | 25. (a) (b) (c) (d) (e) |
| 12. (a) (b) (c) (d) (e) |                         |
| .....                   |                         |
| 13. (a) (b) (c) (d) (e) |                         |
| 14. (a) (b) (c) (d) (e) |                         |

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

### Multiple Choice

1.(6 pts.) Find the limit

$$\lim_{x \rightarrow 0} \frac{1 - \sqrt{x+1}}{x}.$$

- (a)  $-\frac{1}{2}$
- (b) 0
- (c)  $\frac{1}{3}$
- (d)  $-1$
- (e) Does not exist.

2.(6 pts.) The function

$$f(x) = \frac{x^2 + x - 6}{x^2 - 4}$$

has a removable discontinuity at  $x = 2$ . We can remove this discontinuity by defining  $f(2)$  to be

- (a) 1
- (b)  $\frac{3}{2}$
- (c)  $\frac{5}{4}$
- (d)  $\frac{1}{3}$
- (e) 0

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

3.(6 pts.) The linearization of the function  $f(x) = x^3 + 3$  at  $x = -1$  is

- (a)  $y = 3x + 5$
- (b)  $y = 3x^2 + 3$
- (c)  $y = 3x^2(x^3 - 3)$
- (d)  $y^2 = 3x + 1$
- (e)  $y = x + 7$

4.(6 pts.) The second derivative of

$$y = (x + 1)(x - 1)(x^2 + 1)$$

is

- (a)  $24x$
- (b)  $4x^3$
- (c)  $12x^2$
- (d)  $x^2 + 2x - 1$
- (e)  $4x^2 - 2x + 1$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

5.(6 pts.) If

$$f(x) = \frac{(x+1)(x+2)}{(x-1)(x-2)},$$

then  $f'(0)$  is

- (a) 1
- (b) -2
- (c)  $-\frac{1}{4}$
- (d) 3
- (e) 0

6.(6 pts.) If

$$r = \sin \theta + \tan \theta,$$

then  $\frac{dr}{d\theta} =$

- (a)  $\cos \theta + \sec^2 \theta$
- (b)  $\tan \theta - \cos \theta$
- (c)  $\sec \theta + \tan \theta$
- (d)  $\cos \theta + \cot^2 \theta$
- (e)  $\cos \theta + \csc \theta$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

7.(6 pts.) If

$$f(x) = \sqrt{1 + \sqrt{1 + x}},$$

then  $f'(8) =$

(a)  $\frac{1}{8}$

(b)  $\frac{1}{24}$

(c)  $\frac{1}{12}$

(d)  $\frac{1}{9}$

(e)  $\frac{1}{2}$

8.(6 pts.) A body travels in rectilinear motion according to the law

$$s = 20t^2 - 4t^3 - t^4.$$

Where (i.e. what position), after the motion gets started, does the body first come to rest?

(a)  $s=2$

(b)  $s=12$

(c)  $s=32$

(d)  $s=24$

(e)  $s=36$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

9.(6 pts.) The equation of the tangent line to the curve

$$y = x^3 + 6x^2 + 10x + 6$$

at  $x = -2$  is

- (a)  $y = \frac{x}{2}$
- (b)  $x + y - 2 = 0$
- (c)  $x + 2y - 2 = 0$
- (d)  $2x + y + 2 = 0$
- (e)  $2x + y = 0$

10.(6 pts.) If

$$x^2 = \frac{x - y}{x + y},$$

find  $\frac{dy}{dx}$  at the point  $(1, 0)$ .

- (a) 0
- (b) 1
- (c) 2
- (d) -1
- (e)  $-\frac{1}{2}$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

11.(6 pts.) What are the asymptotes (both vertical and horizontal) for the graph of

$$y = \frac{x|x| - 4}{x^2 + 2x - 3}?$$

- (a) There are no asymptotes
- (b)  $y = 2, x = 1$
- (c)  $y = 1, x = 1$
- (d)  $y = 1, x = -3, x = 1$
- (e)  $y = 1, y = -1, x = -3, x = 1$

12.(6 pts.) You are planning to make an open box from a square 6 by 6 inch piece of cardboard by cutting squares from the corners and folding up the sides. What is the largest volume possible for a box constructed in this way

- (a) 8 cu. in.
- (b) 40 cu. in.
- (c) 24 cu. in.
- (d) 32 cu. in.
- (e) 16 cu. in.

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

**13.**(6 pts.) Identify the critical numbers of the function  $f(x) = (x + 4)^5(x - 3)^4$ .

(a)  $x = 3, -\frac{1}{9}$

(b)  $x = 3, -4, -\frac{1}{9}$

(c)  $x = 3, -4$

(d)  $x = 3, -4, -\frac{1}{2}$

(e)  $x = 3, -\frac{1}{2}$

**14.**(6 pts.) A girl flies a kite at a height of 300 feet, the wind carrying the kite horizontally away from her at rate of 25ft/sec. How fast must she let out the string when the kite is 500 feet away from her?

(a) 20 ft/sec

(b) 15 ft/sec

(c)  $75/3$  ft/sec

(d)  $10\sqrt{3}$  ft/sec

(e)  $125/4$  ft/sec

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

**15.**(6 pts.) The graph of the derivative  $f'$  of a function  $f$  is

How many local minima does  $f$  possess?

- (a) 0
- (b) 4
- (c) 2
- (d) 3
- (e) 1

**16.**(6 pts.) Referring to the graph of the last problem, on which of the following intervals is  $f$  concave up?

- (a)  $(3, 5)$
- (b) None.
- (c)  $(0, 2)$
- (d)  $(2, 4)$
- (e)  $(1, 3)$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

17.(6 pts.) The function  $y = f(x)$  satisfies the differential equation

$$\frac{dy}{dx} = 6\sqrt{x+1} - 5$$

with  $f(0) = 5$ . Its value  $f(3)$  is

- (a) 18
- (b) 21
- (c) 15
- (d) 12
- (e) 17

18.(6 pts.) Consider  $\int_1^3 x^3 dx$ . Divide the interval of integration into 5 equal pieces. Which summation below is the Riemann sum for this partition using the right hand end point in each interval?

(a)  $\frac{2}{5} \sum_{i=1}^5 \left(\frac{2i}{5}\right)^3$

(b)  $\frac{2}{5} \sum_{i=0}^4 \left(1 + \frac{2i}{5}\right)^3$

(c)  $\frac{2}{5} \sum_{i=1}^5 \left(1 + \frac{2i}{5}\right)^3$

(d)  $\frac{2}{5} \sum_{i=0}^4 \left(\frac{2i}{5}\right)^3$

(e)  $\frac{1}{5} \sum_{i=1}^5 \left(1 + \frac{2i}{5}\right)^3$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

19.(6 pts.)

$$\int_2^4 x\sqrt{x^2 - 4} dx =$$

- (a)  $12\sqrt{2}$
- (b)  $8\sqrt{3}$
- (c) 8
- (d) 12
- (e)  $4\sqrt{6}$

20.(6 pts.) If  $f(x) = \int_0^{4x} \cos(t^2)dt$ , find  $f'(x)$ .

- (a)  $-4 \cos(16x^2)$
- (b)  $-4 \cos(4x^2)$
- (c)  $-16 \cos(4x^2)$
- (d)  $4 \cos(4x^2)$
- (e)  $4 \cos(16x^2)$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

21.(6 pts.) Evaluate the integral

$$\int_{-\pi/2}^{\pi/2} (\cos x + \sin x^3) dx.$$

(a)  $-\frac{5}{4}$

(b) 0

(c)  $\frac{1}{4}$

(d)  $-\frac{1}{2}$

(e) 2

22.(6 pts.) The curves  $y = x^4 - 3$  and  $y = -x^4 + 5$  enclose an area. Set up a definite integral which calculates the area of this region.

(a)  $\int_0^{\sqrt{3}} (8 - 2x^4) dx$

(b)  $\int_{-\sqrt{2}}^{\sqrt{2}} (8 - 2x^4) dx$

(c)  $\int_{-\sqrt{2}}^{\sqrt{2}} 2 dx$

(d)  $\int_{-1}^1 2 dx$

(e)  $\int_{-1}^1 (8 - 2x^4) dx$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

**23.**(6 pts.) The plane region bounded below by the graph of  $y = x$  and above by the graph  $y = \sqrt{x}$  is rotated about the line  $x = 5$ . Which integral below gives the volume?

- (a)  $2\pi \int_0^1 (5 - x) \cdot (\sqrt{x} - x) dx$
- (b)  $\pi \int_0^1 (5 - \sqrt{x})^2 - (5 - x)^2 dx$
- (c)  $2\pi \int_0^1 (5 - x) \cdot (x - \sqrt{x}) dx$
- (d)  $\pi \int_0^1 (5 - x)^2 - (5 - \sqrt{x})^2 dx$
- (e)  $2\pi \int_0^1 (x - 5) \cdot (\sqrt{x} - x) dx$

**24.**(6 pts.) Consider the plane region bounded by the graphs of  $y = \sqrt{x}$ ,  $y = 0$ ,  $x = 0$  and  $x = 1$ . Rotate this region about the line  $y = -3$  and calculate the volume.

- (a)  $\frac{7\pi}{2}$
- (b)  $\frac{15\pi}{2}$
- (c)  $\frac{9\pi}{2}$
- (d)  $\frac{27\pi}{2}$
- (e)  $\frac{3\pi}{3}$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

**25.**(6 pts.) Find the average of  $f(x) = \sin^2 x \cos x$  over  $[0, \frac{\pi}{2}]$ .

(a)  $\frac{1}{3\pi}$

(b)  $\frac{2}{\pi}$

(c)  $\frac{1}{\pi}$

(d)  $\frac{1}{3}$

(e)  $\frac{2}{3\pi}$

**Math 125, Final Exam**  
**December 16, 2004**

Name: \_\_\_\_\_

Instructor: ANSWERS

- Be sure that you have all 14 pages of the test.
- No calculators are to be used.
- The exam lasts for two hours.
- **When told to begin, remove this answer sheet and keep it under the rest of your test. When told to stop, hand in just this one page.**
- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

- |                         |                         |
|-------------------------|-------------------------|
| 1. (●) (b) (c) (d) (e)  | 15. (a) (b) (●) (d) (e) |
| 2. (a) (b) (●) (d) (e)  | 16. (a) (b) (c) (●) (e) |
| .....                   |                         |
| 3. (●) (b) (c) (d) (e)  | 17. (●) (b) (c) (d) (e) |
| 4. (a) (b) (●) (d) (e)  | 18. (a) (b) (●) (d) (e) |
| .....                   |                         |
| 5. (a) (b) (c) (●) (e)  | 19. (a) (●) (c) (d) (e) |
| 6. (●) (b) (c) (d) (e)  | 20. (a) (b) (c) (d) (●) |
| .....                   |                         |
| 7. (a) (●) (c) (d) (e)  | 21. (a) (b) (c) (d) (●) |
| 8. (a) (b) (●) (d) (e)  | 22. (a) (●) (c) (d) (e) |
| .....                   |                         |
| 9. (a) (b) (c) (●) (e)  | 23. (●) (b) (c) (d) (e) |
| 10. (a) (b) (c) (●) (e) | 24. (a) (b) (●) (d) (e) |
| .....                   |                         |
| 11. (a) (b) (c) (d) (●) | 25. (a) (b) (c) (d) (●) |
| 12. (a) (b) (c) (d) (●) |                         |
| .....                   |                         |
| 13. (a) (●) (c) (d) (e) |                         |
| 14. (●) (b) (c) (d) (e) |                         |