Hyperfine Interactions **146/147:** 67–70, 2003.

Vacuum-Polarization Corrections to Parity-Nonconserving Effects in Atomic Systems

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Abstract. The dominant one-loop vacuum-polarization correction to atomic wave functions are evaluated for the $6s$–$7s$ parity nonconserving (PNC) E1 transition amplitude in cesium. This correction increases the size of the PNC amplitude by 0.4% and thus increases the difference between the experimental value of the weak charge Q_W and the one predicted by the standard model.

Key words: parity non-conservation, atomic systems.

1. Introduction

Parity non-conservation (PNC) in atomic systems as described within the standard model of the electroweak interaction via exchange of Z bosons between bound electrons and nuclear quarks leads to non-vanishing electric-dipole matrix elements between atomic states with the same parity. The nuclear spin-independent part of PNC matrix elements arising from the nuclear vector current is proportional to the conserved weak charge Q_W and thus is sensitive to possible "new physics" beyond the standard model such as the existence of additional neutral Z' bosons or new heavy fermions.

Measurements of the $6s$–$7s$ PNC amplitude in ^{133}Cs were carried out at the level of accuracy of about 2% by Gilbert and Wieman [1], and at about 0.3% accuracy by Wood *et al.* [2], respectively. Bennett and Wieman [4] have analyzed the differences between experimental and theoretical values deduced for transition amplitudes in ^{133}Cs and concluded that the error in the atomic-structure calculations which enter in the PNC amplitude should be reduced from 1% (this value given in [3]) to 0.4%. As a result they found that the experimentally deduced value for the weak charge in ^{133}Cs deviates from the standard-model value [5]

$$Q_W\left(^{133}\mathrm{Cs}\right) = -73.09 \pm 0.03 \tag{1}$$

by 2.3σ. Such a difference between experiment and theory suggests, e.g., the existence of a second neutral Z' boson [6].

It has been shown by Derevianko [7] that Breit corrections to the PNC amplitude decrease the magnitude of the calculated PNC amplitude by about 0.6%. As a consequence the Breit corrections reduce the deviation from the standard model to 1.6σ when assigning an error of about 0.4% to the calculated matrix elements as it has been assumed in [4]. Taking a more conservative value of a 1% error as given in [3] leads to a deviation of only about 0.9σ.

Radiative corrections to PNC matrix elements $\langle np_{1/2}|H_{\mathrm{PNC}}|n's_{1/2}\rangle$ in the strong Coulomb field of a heavy one-electron ion were considered recently by Bednyakov *et al.* [10]. These corrections were decomposed into two parts, radiative corrections to the operator H_{PNC} and radiative corrections to the wave functions $|n's_{1/2}\rangle$ and $|np_{1/2}\rangle$. We have found that these corrections increase the size of the PNC amplitude in ^{133}Cs by about 0.4% and, correspondingly, increase the difference between the theoretical and experimental weak charge. Moreover we found that these corrections are insensitive to electron–electron correlation effects.

2. One-loop radiative corrections to PNC effects

One-loop vacuum polarization corrections in an external Coulomb field modify the Coulomb interaction at short distances. To lowest order in $Z\alpha$ this modification is described by the Uehling potential for an extended nuclear charge distribution $\rho(r)$ [14]:

$$\delta V(r) = -\frac{2\alpha^2}{3r}\int_0^\infty \mathrm{d}x\, x\, \rho(x)\int_1^\infty \mathrm{d}t\sqrt{t^2-1} \times\left(\frac{1}{t^3}+\frac{1}{2t^5}\right)\left(\mathrm{e}^{-2ct|r-x|}-\mathrm{e}^{-2ct(r+x)}\right). \tag{2}$$

Here we employ atomic units, where $\alpha = 1/137.036\ldots$ is the fine-structure constant and $c \equiv \alpha^{-1}$ is the speed of light. We have carried out calculations of the $6s$–$7s$ PNC amplitude in the presence of the additional Uehling potential. The first calculation is performed at the level of the "weak" Dirac–Hartree–Fock (DHF) approximation. The perturbation $\delta\psi_v^{\mathrm{DHF}}$ to a valence electron wave function ψ_v^{DHF} induced by the *weak* interaction h_{PNC} satisfies the inhomogeneous DHF equation

$$\left(h_0 + V^{\mathrm{DHF}} - \epsilon_v^{\mathrm{DHF}}\right)\delta\psi_v^{\mathrm{DHF}} = -h_{\mathrm{PNC}}\psi_v^{\mathrm{DHF}}. \tag{3}$$

In this equation, V_{HF} is the HF potential of the closed xenon-like core and ϵ_v^{HF} is the eigenvalue of the unperturbed DHF equation. The PNC amplitude is then given by the sum of two terms:

$$E_{\mathrm{PNC}} = \left\langle\psi_{7s}^{\mathrm{HF}}\middle|D\middle|\delta\psi_{6s}^{\mathrm{HF}}\right\rangle + \left\langle\delta\psi_{7s}^{\mathrm{HF}}\middle|D\middle|\psi_{6s}^{\mathrm{HF}}\right\rangle, \tag{4}$$

where D denotes the dipole operator. In Table I, we provide numerical results for the individual terms of Equation (4) calculated with and without the additional Uehling potential δV of Equation (2), respectively. This one-loop radiative correction increases the magnitude of the sum of both contributions by 0.41%.

A second calculation has been performed in the "weak" random-phase approximation (RPA) which accounts for the class of correlation corrections associated

Table I. One-loop corrections to the weak PNC amplitude for the $6s$–$7s$ E1 transition in ^{133}Cs. Units: $iea_0 \times 10^{-12}(-Q_W/N)$

Type	$\langle\phi_{7s}\|D\|\delta\phi_{6s}\rangle$	$\langle\delta\phi_{7s}\|D\|\phi_{6s}\rangle$	$E_{\rm PNC}$
Weak HF approximation:			
DHF	2.749183	−10.14387	−7.394685
DHF + δV	2.760414	−10.18581	−7.425391
Δ (%)	0.41	0.41	0.41
Weak RPA approximation:			
RPA	3.457036	−12.72562	−9.268581
RPA + δV	3.471169	−12.77834	−9.307166
Δ (%)	0.41	0.41	0.41

with weak perturbations of the core orbitals. This class of correlation corrections is included by solving

$$\left(h_0 + V^{\rm HF} - \epsilon_v^{\rm HF}\right)\delta\psi_v^{\rm RPA} = -\left[h_{\rm PNC} + V_{\rm PNC}^{\rm HF}\right]\psi_v^{\rm PRA}, \tag{5}$$

where $V_{\rm PNC}^{\rm HF}$ is the weak correction to the HF potential. The resulting PNC amplitude is given by

$$E_{\rm PNC} = \left\langle\psi_{7s}^{\rm HF}\middle|D\middle|\delta\psi_{6s}^{\rm RPA}\right\rangle + \left\langle\delta\psi_{7s}^{\rm RPA}\middle|D\middle|\psi_{6s}^{\rm HF}\right\rangle. \tag{6}$$

In Table I we also compare the corresponding results with and without the Uehling corrections. Again, the vacuum-polarization correction is seen to increase the results by 0.41%. From these calculations we can conclude that vacuum-polarization corrections are independent from electron–electron correlation effects.

3. Conclusion

One-loop vacuum-polarization corrections to the electron wave functions have been evaluated. The resulting wave functions are used to calculate the $6s$–$7s$ PNC transition amplitude in ^{133}Cs. They lead to an increase of the magnitude of the amplitude by 0.4%. Taking into account the Breit and the one-loop vacuum-polarization corrections we deduce the following value for the weak charge

$$Q_W^{\rm expt}\left(^{133}{\rm Cs}\right) = -72.12 \pm (0.28)_{\rm exp} \pm (0.34)_{\rm theor}.$$

This value differs by 2.2σ from the standard model. If we assume an error of about 1% in the theoretical amplitude, then the error in the theoretical component of $Q_W^{\rm exp}$ increases to $\pm(0.74)_{\rm theor}$ and the deviation from the value predicted by the standard model becomes 1.2σ.

Acknowledgements

The work of W.R.J. was supported in part by an Alexander von Humboldt Award from the Federal Republic of Germany and in part by the U.S. National Science Foundation under grant No. 99-70666. W.R.J. also owes a debt of gratitude to O. Sushkov for discussions on this problem. G.S. acknowledges support from DFG, GSI and BMBF.

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