

Basic Combinatorics (Math 40210) Sec 01, Spring 2014, Quiz 3
redo

Name:

April 15, 2014

1. A sequence is defined recursively by the rules $a_0 = 0$ and $a_{k+1} = a_k + 2^k$ for $k \geq 0$. Show that the generating function $A(x) = a_0 + a_1x + a_2x^2 + \dots$ is

$$A(x) = \frac{x}{(1-x)(1-2x)}.$$

2. A subset of $\{1, \dots, n\}$ is called *well-spaced* if for any two distinct elements x, y in the set, $|x - y| \geq 3$ (so, for example, $\{1, 4, 8, 11\}$ is well spaced, but $\{6, 8, 11\}$ is not, because $|6 - 8| = 2$). Let s_n be the number of subsets of $\{1, \dots, n\}$ that are well-spaced. For example, $s_0 = 1$ (because the empty set is well spaced), and $s_4 = 6$ (the six well-spaced sets being the empty set, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$ and $\{1, 4\}$). Find a recurrence relation that calculates s_n in terms of some smaller values, and give enough initial conditions to fully specify the sequence. (**Hint:** Is n in the subset, or not?). Check that your recurrence gives $s_6 = 13$ (and if not, re-do your answer!)