Basic Combinatorics

Math 40210, Section 01 — Fall 2012

Homework 5 — due Friday October 5

General information: Homework is an essential part of your learning in this course, so please take it very seriously. It is extremely important that you keep up with the homework, as if you do not, you may quickly fall behind in class and find yourself at a great disadvantage during exams.

You should treat the homework as a learning opportunity, rather than something you need to get out of the way. Reread, revise, and polish your solutions until they are correct, concise, efficient, and elegant. This will really deepen your understanding of the material. I encourage you to talk with your colleagues about homework problems, but your final write-up must be your own work.

You should present your final homework solutions clearly and neatly. Keep in mind that when you write a homework solution, you are trying to communicate the solution to someone other than yourself, so incomplete sentences and personal shorthand is not helpful!

Due to manpower issues, I will only grade selected homework problems, but I plan to quickly post solutions to all the problems soon after I've collected them up.

Reading:

- Section 1.5.2
- Section 1.5.3
- Section 1.5.4 (The sketch of the proof of Thm 1.40 will not be examined)
- Section 1.6.1
- Section 1.6.2, excluding the proof of Theorem 1.43
- Section 1.6.3

Problems:

- Section 1.5.2: 1, 4, 6, 7, 9 (for 9, here's what you are being asked: show that there is a graph on n vertices with q edges that has $q \leq 3n - 6$, but has no planar representation)
- Section 1.5.4: 2
- Section 1.6.1: 1(a,c,d,e), 2, 4, 6 (for 1c), the *complete multipartite graph* $K_{r_1,r_2,\ldots,r_t}$ has vertex set $R_1 \cup R_2 \cup \ldots \cup R_t$, where $|R_i| = r_i$ for each i, the R_i's are disjoint from each other, and there is an edge from u to v if and only if u and v are in *different* R_i's)
- Section 1.6.2: 1, 4, 6, 8, plus the following additional question:
  - For each even $n \geq 4$, give an example of a graph G_n on n vertices with $n/\alpha(G_n) = 2$ (i.e., $\alpha(G_n) = n/2$) and $\chi(G_n) = n/2$ (this shows that the bound $\chi(G) \geq n/\alpha(G)$ can sometimes be very wrong)
- Section 1.6.3: 2