Basic Combinatorics

Math 40210, Section 01 — Fall 2012

Homework 1 — due Friday, August 31

General information: Please write you name and the course number or name at the top of the first page of your solutions, and please staple together all pages.

Homework is an essential part of your learning in this course, so please take it very seriously. It is extremely important that you keep up with the homework, as if you do not, you may quickly fall behind in class and find yourself at a great disadvantage during exams.

You should treat the homework as a learning opportunity, rather than something you need to get out of the way. Reread, revise, and polish your solutions until they are correct, concise, efficient, and elegant. This will really deepen your understanding of the material. I encourage you to talk with your colleagues about homework problems, but your final write-up must be your own work.

You should present your final homework solutions clearly and neatly. Keep in mind that when you write a homework solution, you are trying to communicate the solution to someone other than yourself, so incomplete sentences and personal shorthand is not helpful!

Due to manpower issues, I will only grade selected homework problems, but I plan to quickly post solutions to all the problems soon after I've collected them up.

Reading:

- Introduction to Chapter 1
- Section 1.1.1
- Section 1.1.2, up to the end of the paragraph that begins on the bottom of page 7
- Section 1.1.3
- Introduction to Section 1.2
- Section 1.2.1 (the proof of Theorem 1.6 is optional)

Problems:

- Section 1.1.1: 1, 4 (for 4, you have to have read the introduction to Chapter 1 to know what the problem is)
- Section 1.1.2: 1, 2, 5, 6, 10, 16 (once you've done 6, youll have completed our proof that a graph is bipartite if and only if it has no odd cycles. BTW, the book doesnt actually define "closed walk", but its what you think it is: a walk in which $v_1 = v_k$, that is, a walk that starts and ends at the same place.)
- Section 1.1.3: 3, 8, 10
- Section 1.2.1: 2, 3 (*just* for P_{2k}, P_{2k+1}), 7, ~~8(d), 11(a)~~ (moved to homework 2)

Here are two problems that **are** part of the homework assignment, but are not from the textbook:

- Extra problem 1: 1.1.2(6) asks to show that every odd length closed walk contains an odd cycle. Prove or give a counterexample to the analogous statement with "odd" replaced by "even": every even length closed walk contains an even cycle.
- Extra problem 2: How many different bipartitions does a bipartite graph G have into a first partite set X and a second partite set Y? Here, I say that a bipartition $X \cup Y$ is *different* from $U \cup V$ if at least one of $X \neq U$, $Y \neq V$ holds; so in particular the bipartition $X \cup Y$ is different from the bipartition $Y \cup X$ (i.e., the order in which I name the partite sets matters). Your answer to this question should depend on some parameter of the graph G that you may need to define (for example, "$(\Delta(G) - \tau(G))^2$ where $\tau(G)$ is the length of the longest path in G" is the sort of answer I'm expecting).