Math 30530, Probability

Quiz 5, Friday April 12

Solutions

A bag has three balls, numbered "1", "2" and "3". I draw twice from the bag, *without replacement*, and note the two numbers that I see. Let X be the smaller of the two numbers, and Y the larger.

1. Find the joint mass function of X and Y.

   Solution: There are 6 equally likely possibilities:

   - first 1, then 2; or first 2, then 1: both lead to $X = 1, Y = 2$;
   - first 1, then 3; or first 3, then 1: both lead to $X = 1, Y = 3$;
   - first 2, then 3; or first 3, then 2: both lead to $X = 2, Y = 3$;

   so $p_{X,Y}(1,2) = 1/3$, $p_{X,Y}(1,3) = 1/3$, and $p_{X,Y}(2,3) = 1/3$, with all other probabilities being 0.

2. Write down the marginal densities of X and Y.

   Solution: For X: $p_X(1) = 2/3, p_X(2) = 1/3, p_X(3) = 0$.

   For Y: $p_Y(1) = 0, p_Y(2) = 1/3, p_Y(3) = 2/3$.

3. Are X and Y independent? Briefly say why or why not.

   No. It is not the case that $p_X(x)p_Y(y) = p_{X,Y}(x,y)$. For example, $p_X(2)p_Y(2) = 1/9 \neq 0 = p_{X,Y}(2,2)$.

4. What is the expected value of XY?

   Solution: $E(XY) = (1)(2)(1/3) + (1)(3)(1/3) + (2)(3)(1/3) = 11/3$.

5. Compute $\mathrm{Cov}(X,Y)$, and give a very brief interpretation of your answer.

   Solution: $E(X) = (1)(2/3) + (2)(1/3) + (3)(0) = 4/3$ and $E(Y) = (1)(0) + (2)(1/3) + (3)(2/3) = 8/3$. So

   $$\mathrm{Cov}(X,Y) = E(XY) - E(X)E(Y) = \frac{11}{3} - \left(\frac{4}{3}\right)\left(\frac{8}{3}\right) = \frac{1}{9}.$$

   So X, Y are *positively* correlated — they tend to be large together or small together.