Math 30530, Probability

Quiz 3, Wednesday March 19

Solutions

1. I draw cards from a standard 52-card deck, with replacement, stopping the fourth time I select an ace.

   (a) What's the probability that I stop after 20 draws? (An unsimplified answer involving factorials, binomial coefficients, et cetera, is fine.)
   Solution: Model X, the number of draws, by NegBin$(4, 1/13)$. $P(X = 20) = \binom{19}{3}(1/13)^4(12/13)^{16}$.

   (b) What's the expected number of cards I draw before I stop? (Exact numerical answer required.)
   Solution: $E(X) = 4/(1/13) = 52$.

2. A box of 200 bulbs has 20 that are defective. I take 12 bulbs from the box at random. Calculate

   (a) the probability that at least 11 of them work. (An unsimplified answer involving factorials, binomial coefficients, et cetera, is acceptable.)
   Solution: Model Y, the number of bulbs that work, by HGeom$(180, 20, 12)$.

   $$P(X \geq 11) = \frac{\binom{180}{11}\binom{20}{1} + \binom{180}{12}\binom{20}{0}}{\binom{200}{12}}.$$

   (b) the expected number in the sample of 12 that work. (Exact numerical answer required.)
   Solution: $E(Y) = 12\left(\frac{180}{200}\right) = 10.8$.

3. Historically, Notre Dame has admitted on average 2.5 Irish students per year. Using an appropriate probability distribution, estimate the probability (exact numerical answer in both cases) that

   (a) there will be at least one Irish student admitted next year, and
   Solution: Model Z, the number of Irish students, by Poisson(2.5). $P(Z \geq 1) = 1 - P(Z = 0) = 1 - e^{-2.5} \approx 0.918$.

   (b) there will be at least one Irish student admitted each year, for each of the next six years.
   Solution: Assuming independence from year to year, from the previous part this is $(1 - e^{-2.5})^6 \approx 0.598$.