

# Paradox of the two children

Math 30530, Fall 2013

September 3, 2013

## Scenario 1: “The elder is a girl”

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$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{1/4}{1/2} = \frac{1}{2}$$

**OR**

$$\Pr(A|B) = \frac{|A \cap B|}{|B|} = \frac{1}{2}$$

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$$\Pr(A|C) = \frac{\Pr(A \cap C)}{\Pr(C)} = \frac{1/4}{3/4} = \frac{1}{3}$$

**OR**

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