Math 30530 — Introduction to Probability

Quiz 3 – Wednesday October 9, 2013

Solutions

1. I have five M&M candies in a bag; three are red and two are green. I take out three at random, and count how many red ones I selected. Let X be this number. Calculate the expected value of X.

   Solution: Possible values for X are $1, 2$ and 3 (I can't select 0) reds. The probability of selecting 1 red (and 2 greens) is $\binom{3}{1}\binom{2}{2}/\binom{5}{3} = .3$. The probability of selecting 2 reds (and 1 green) is $\binom{3}{2}\binom{2}{1}/\binom{5}{3} = .6$. The probability of selecting 3 reds (and 0 greens) is $\binom{3}{3}\binom{2}{0}/\binom{5}{3} = .1$. So

   $$E(X) = 1(.3) + 2(.6) + 3(.1) = 1.8.$$

2. Calculate the variance and standard deviation of X.

   Solution:

   $$E(X^2) = 1^2(.3) + 2^2(.6) + 3^2(.1) = 3.6,$$

   so $\text{Var}(X) = 3.6 - (1.8)^2 = .36$ and $\sigma = .6$.

3. Instead of eating the three candies, I decide to sell them. I can sell x red M&M's for x^3 dollars (red M&M's are more valuable when they are gathered in groups), and I can sell each green M&M for 1 dollar. What is my expected revenue from selling the three M&M's?

   Solution: If I have x red M&M's then I have $3 - x$ greens, and I can sell for $x^3 + 3 - x$ dollars. So we are looking for $E(X^3 - X + 3)$. This is

   $$E(X^3 - X + 3) = (1^3 - 1 + 3)(.3) + (2^3 - 2 + 3)(.6) + (3^3 - 3 + 3)(.1) = 3(.3) + 9(.6) + 27(.1) = 9.$$

   Notice that this is **not** the same as $E(X)^3 - E(X) + 3$: expectation is linear, so $E(aX + b) = aE(X) + b$, but in general it is not true that $E(f(X)) = f(E(X))$ for a non-linear function f.