

Math 30530 — Introduction to Probability

Quiz 4 – Wednesday October 31, 2012

SOLUTIONS

A pair of random variables (X, Y) have joint density

$$f_{X,Y}(x, y) = \begin{cases} \frac{3}{8}(x^2 + y) & \text{if } 0 \leq x \leq 1, 0 \leq y \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

1. Compute the marginal density of X .

Solution: If $x < 0$ or $x > 1$ the $f_X(x) = 0$. For $0 \leq x \leq 1$ we have

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x, y) dy = \int_0^2 \frac{3}{8}(x^2 + y) dy = \frac{3}{8}(2x^2 + 2).$$

So

$$f_X(x) = \begin{cases} \frac{3}{8}(2x^2 + 2) & \text{if } 0 \leq x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

2. Write down, but don't evaluate, an integral whose value is $\Pr(X + Y \geq 2)$.

Solution: Either

$$\int_{x=0}^1 \int_{y=2-x}^2 \frac{3}{8}(x^2 + y) dy dx$$

or

$$\int_{y=1}^2 \int_{x=2-y}^1 \frac{3}{8}(x^2 + y) dx dy$$

would work.