Introduction to Probability

Math 30530, Section 01 — Fall 2012

Homework 8 — due Friday November 16

General information: Homework is an essential part of your learning in this course, so please take it very seriously. It is extremely important that you keep up with the homework, as if you do not, you may quickly fall behind in class and find yourself at a disadvantage during exams.

You should treat the homework as a learning opportunity, rather than something you need to get out of the way. Reread and revise your solutions until they are correct and concise. This will help deepen your understanding of the material. I encourage you to talk with your colleagues about homework problems, but your final write-up must be your own work.

You should present your final homework solutions clearly and neatly. Keep in mind that when you write a homework solution, you are trying to communicate the solution to someone other than yourself, so incomplete sentences and personal shorthand is not helpful!

I plan to quickly post solutions to all the problems after I've collected them up.

Reading:

- Chapter 29
- Chapter 30
- Chapter 31
- Chapter 32
- Chapter 35

Problems: (**GW** indicates that the problem is taken from the course textbook by Gundlach and Ward)

Many of the problem from the relevant chapters of **GW** are fairly routine. It's very valuable to do a lot of these problems, but I'll leave it up to you to decide how many or which ones you want to do. Here's a list of possible problems, **none of which should be submitted for grading**, but which you should do if you feel you need practice on the various topics we've covered. Feel free also to ask about these problems at office hours.

- Chapter 29: 2, 6, 9 through 17, 18 through 24.
- Chapter 31: 3 through 7, 9, 10, 14 through 16.
- Chapter 32: 2 through 7, 9 through 12.
- Chapter 35: 1 through 9, 15 through 22.

Here are the questions that should be **submitted**:

1. **GW** 29.1a)
2. **GW** 29.4
3. **GW** 29.24
4. **GW** 31.11
5. **GW** 32.8
6. **GW** 32.14
7. **GW** 32.17
8. **GW** 35.28
9. I break a piece of chalk that is 6 inches long into three pieces, by picking two spots uniformly and independently at which to make breaks. What is the probability that the three pieces can be combined to form a triangle?
10. Let X be exponential with parameter λ. We know that $E(X) = 1/\lambda$ and $E(X^2) = 2/\lambda^2$.
    (a) Find $E(X^3)$ (for this, you might think that you have to do integration by parts three times; but after the first integration by parts, you should be able to use the fact that you know that $E(X^2) = 2/\lambda^2$ to finish the problem without any more integration).
    (b) Find $E(X^4)$ (again, one integration by parts should do, together with your result from the last part).
    (c) Guess what is $E(X^n)$, based on the pattern from the previous answers.

(d) If you are familiar with proof by induction, show that your guess is correct. (If not, just ignore this part.)

11. Let X be a uniform random number between 0 and 1. and let $0 < a < b$ be constants.

    (a) Find the distribution of $(b-a)X + a$.

    (b) Find the density function of $(b-a)X + a$.

    (c) Compare your answer to the last part with the density function of a uniform random number between a and b. Are they the same? (**Hint**: yes they are, unless you've made a computation error. In general, if $X \sim \text{Uniform}(0,1)$ then $(b-a)X + a \sim \text{Uniform}(a,b)$.)

12. Someone calls the police to tell them that a bomb has been left in a public place, and that it will off at a random time that is an exponential random variable with expected value 2 hours. The police contact the bomb squad, who tells them that it will take them a quarter of an hour to get to the site of the bomb, and that they can then diffuse it in a time that is an exponential random variable with expected value half an hour.

    (a) What is the probability that the bomb will go off before the bomb squad arrives?

    (b) The bomb squad arrives, and the bomb has not yet gone off. What is the probability, if the bomb squad does nothing, that it goes off in the next quarter of an hour?

    (c) The moment that the bomb squad has been contacted, a reporter calls the police and asks what is the probability that the bomb will be successfully diffused. What should the police reply?

13. A coffee dispenser releases into a 10 ounce cup an amount of coffee that is normally distributed with mean μ ounces (this value can be set by the operator by turning a dial) and standard deviation .5 ounces.

    (a) If μ is set to 9.5, what is the probability that the coffee will overflow the cup?

    (b) Customers are unhappy when their cups have fewer than 8.8 ounces of coffee in them. What proportion of customers are unhappy when μ is set to 9.5?

    (c) What value should the operator set μ to be, to be sure that only 5% of cups overflow?

    (d) One day the operator discovers that the "μ" dial is jammed at $\mu = 9.5$. He can adjust σ by altering the tightness of a screw on the back of the dispenser. What value of σ should he select so that only 5% of cups overflow?