# Introduction to Probability

## Math 30530, Section 01 — Fall 2012

## Homework 4 — due Monday September 24

**General information**: Homework is an essential part of your learning in this course, so please take it very seriously. It is extremely important that you keep up with the homework, as if you do not, you may quickly fall behind in class and find yourself at a disadvantage during exams.

You should treat the homework as a learning opportunity, rather than something you need to get out of the way. Reread and revise your solutions until they are correct and concise. This will help deepen your understanding of the material. I encourage you to talk with your colleagues about homework problems, but your final write-up must be your own work.

You should present your final homework solutions clearly and neatly. Keep in mind that when you write a homework solution, you are trying to communicate the solution to someone other than yourself, so incomplete sentences and personal shorthand is not helpful!

I plan to quickly post solutions to all the problems after I've collected them up.

**Reading**:

- Chapter 8
- Chapter 9
- Chapter 10
- Chapter 11

**Problems**: (**GW** indicates that the problem is taken from the course textbook by Gundlach and Ward)

1. **GW** 8.2
2. **GW** 8.3
3. **GW** 8.7 (for this one, also find the joint mass of $X$ and $Y$)
4. **GW** 8.10 (part a) only)
5. **GW** 9.7
6. **GW** 9.8
7. **GW** 9.10
8. **GW** 9.18
9. **GW** 10.6
10. **GW** 10.15 (part a) only)
11. **GW** 10.23
12. I'm on *Who wants to be a Millionaire*, looking at the \$250,000 question. My pot of money is currently \$100,000. If I get the \$250,000 question right, my pot goes to \$250,000, and I get the chance to look at the \$500,000 question; if I get that right, my pot goes to \$500,000, and I get the chance to look at the \$1,000,000 question; if I get that right, my pot goes to \$1,000,000, and the game stops. If ever I get a question wrong, my pot goes down to \$25,000 and the game stops. At any point, I can also choose to not answer a question; in that case, I keep the pot I have at that point, and the game stops.

    I estimate that I will have a 5% chance of knowing the answer to the \$500,000 question, and a 1% chance of knowing the answer to the \$1,000,000 question; in either of these cases, I will not guess the answer unless I know it, and I will answer it if I know it.

    Looking at the \$250,000 question, I have a hunch about the right answer, and estimate that my hunch is correct with probability $p$.

    (a) What is the expected value of my pot, as a function of $p$, if I follow my hunch and give what I think is the right answer? (I'm asking here for expected value of the pot, when the whole game is over: if I answer the \$250,000 question correctly, there is some chance that I might also answer the \$500,000 question correctly, and even the \$1,000,000 question.)

    (b) How sure should I be of my hunch (what's the value of $p$), to have an expected final pot of more than \$100,000 if I go for it on the question I'm looking at?

13. If $X$ is a random variable that takes on values 0, 1, 2 etc. with probabilities $p_0$, $p_1$, $p_2$, etc. (and takes on no other values), show that

$$\mathbf{E}(X) = \Pr(X \geq 1) + \Pr(X \geq 2) + \Pr(X \geq 3) + \ldots = \sum_{k=1}^{\infty} \Pr(X \geq k).$$

14. Three busses come into a bus depot at the same time. Bus $A$ has 40 passengers, bus $B$ has 45 and bus $C$ has 60. Jack and Jill want to do an experiment to estimate the average number of passengers on an incoming bus. Jack picks a bus at random, each bus equally likely, and looks at the number of people in the chosen bus; let $X$ be that number. Jill picks a random passenger from among all three busses, all passengers equally likely, and looks at the number of people who were on that passenger's bus; let $Y$ be that number.

    (a) Compute $\mathbf{E}(X)$ and $\mathbf{E}(Y)$.

    (b) Give a brief explanation as to why Jill's answer should be larger than Jack's, no matter what the number of busses or the number of passengers on each bus.

15. Alice and Bob play the game of Two-finger Morra. Here's how they play: each closes their eyes and holds up either one or two fingers, and at the same time calls out their guess as to how many fingers their opponent is holding up. They then both open their eyes. If exactly one of the two have guessed correctly, (s)he wins a number of dollars equal to the sum of the number of fingers the two players are holding up. If both guess correctly, or neither does, no money changes hands. If both players play independently, and are equally likely to choose each of their four options (one finger up and guess one, one finger up and guess two, two fingers up and guess one, two fingers up and guess two), compute $\mathbf{E}(X)$, where $X$ is how much Alice wins in a playing of the game.

16. Here's a strategy for roulette: bet \$1 on red (which has probability 18/38 of coming up). If red comes up, take your \$2 reward and quit. If red does not come up, make two more \$1 bets on red, then quit. Let $X$ be the total net winnings at the end of this process. $X$ has the following possible values: $+1$ (if red comes up on first roll, or if red does not come up on the first roll, but does on each of the other two), $-1$ (if red does not come up on the first roll, and comes up exactly once during the other two), and $-3$ (if red does not come up on the first roll, and does not on each of the other two either).

    (a) Compute $\Pr(X > 0)$

    (b) Compute $\mathbf{E}(X)$

    (c) Does this seem like a reasonable strategy?

17. Roger Federer and Andy Murray play a best-of-five set tennis match (stopping when one of the two has reached 3 sets), with each of the two equally likely to win each set, independently of the others. Let $X$ be the number of sets they play.

(a) Compute $\mathbf{E}(X)$

(b) If they play ten best-of-five-set matches, each independent of the others, what's the expected total number of sets played? (Justify your answer)

18. Each week I have office hours. There is a probability .05 that my cell-phone will ring during my office hours, while I'm answering a student's question. The semester has 15 weeks. Let $X$ be the number of weeks in which my phone rings during office hours while I'm answering a questions. (Assume weeks are independent of each other).

    (a) What are the possible values for $X$?

    (b) What is the probability that $X = 3$?

    (c) What is the expected value of $X$? (**Hint**: rather than using the direct definition of expectation, break $X$ down into the sum of simpler random variables)