Introduction to Probability

Math 30530, Section 01 — Fall 2012

Homework 1 — due Friday, August 31

General information: Please write you name and the course number or name at the top of the first page of your solutions, and please staple together all pages.

Homework is an essential part of your learning in this course, so please take it very seriously. It is extremely important that you keep up with the homework, as if you do not, you may quickly fall behind in class and find yourself at a disadvantage during exams.

You should treat the homework as a learning opportunity, rather than something you need to get out of the way. Reread and revise your solutions until they are correct and concise. This will help deepen your understanding of the material. I encourage you to talk with your colleagues about homework problems, but your final write-up must be your own work.

You should present your final homework solutions clearly and neatly. Keep in mind that when you write a homework solution, you are trying to communicate the solution to someone other than yourself, so incomplete sentences and personal shorthand is not helpful!

I plan to quickly post solutions to all the problems after I've collected them up.

Reading: The textbook is filled with lots of illustrative examples of the key topics of probability (many more than we can cover in class). Reading the text, preferably with pen and notepaper in hand to verify computations and fill in blanks in arguments, is an extremely important part of the learning process!

- Chapter 1
- Chapter 2
- Chapter 3
- Sections 4.1 and 4.2

Problems: (**GW** indicates that the problem is taken from the course textbook by Gundlach and Ward)

1. A box contains four candy bars: two Mars bars, a Snickers and a Kit-Kat. I randomly draw a bar from the box and eat it, then draw a second and eat that, too. I record the type of bar I ate first, and the type I ate second.

   (a) List all the outcomes in the sample space of this experiment.

   (b) Describe two different events in this experiment, *in words*.

   (c) For each of the two events you described in the last part, describe them by listing all their outcomes.

   (d) Are the two events you described mutually exclusive?

2. I have five keys in my pocket, each of which is a different colour (white, black, gold, silver, yellow). I repeatedly reach into my pocket and pull out a key at random, note its colour, and return it to my pocket. I keep doing this until I draw the gold key, at which point the experiment stops. (The gold key is the one I need to get into my house and out of the rain).

   (a) List all the outcomes in the sample space of this experiment.

   (b) Let E_n be the event that it takes me n or fewer trials to get the gold key. List all the outcomes in E_n. (You may need to use the set notation

   $$\{\text{things}|\text{these conditions are satisfied}\}$$

   here; see page 21.)

   (c) Describe in words what it means for the event $(\cup_{n=1}^{\infty} E_n)^c$ to occur.

   (d) Are E_7 and E_8 mutually exclusive?

3. A and B are mutually exclusive, with $\Pr(A) = .3$ and $\Pr(B) = .5$. What is the probability that

   (a) either A or B occurs?

   (b) A occurs but not B?

   (c) both A and B occur?

4. A total of 28% of American men smoke cigarettes, 7% smoke cigars, and 5% smoke both cigars and cigarettes.

   (a) What percentage smoke neither cigarettes nor cigars?

   (b) What percentage smoke cigars but not cigarettes?

5. The dice in my right hand has three sides painted blue, one side painted red, and two sides painted green. The dice in my left hand has two sides painted blue, two sides painted red, and two sides painted green. I roll the two dice and record the two colours that come up (listing the right dice first).

(a) List the outcomes in the sample space.

(b) By using the fact that the outcomes of the two rolls are independent of each other, calculate the probabilities that i) the two dice both come up red, ii) the two dice show the same colour.

6. Use a Venn diagram to explain why the following relations between events are always true:

   (a) $E \cap F \subset E \subset E \cup F$.

   (b) If $E \subset F$ then $F^c \subset E^c$.

7. Let E, F, G be three events. Using $\cup$, $\cap$ and ^{c}, find expressions for the following events:

   (a) Only E occurs.

   (b) At least one of the events occurs.

   (c) Both E and G, but not F, occur.

   (d) Exactly two of the three events occur.

8. Show that for any events E and F, $\Pr(E \cap F) \geq \Pr(E) + \Pr(F) - 1$.

9. Show that for any events A and B, the probability that exactly one of them occur is $\Pr(A) + \Pr(B) - 2\Pr(A \cap B)$.

10. **GW** 1.1 (there are many right answers to this one)

11. **GW** 1.3

12. **GW** 1.8

13. **GW** 1.12

14. **GW** 2.3

15. **GW** 2.11

16. **GW** 2.19

17. **GW** 2.27

18. **GW** 2.29

19. **GW** 2.33a

20. **GW** 2.37

21. **GW** 3.5

22. **GW** 3.7

23. **GW** 3.10

24. **GW** 3.12