Math 30530 — Introduction to Probability

Quiz 1 – Wednesday August 31, 2011

NAME: _____ *Solution* _____

1. Let E and F be events in a probability space with $P(E) = p$, $P(F) = q$ and $P(E \cup F) = r$.

   (a) (3 pts) Express $P(EF)$ in terms of p, q and r.
   Solution: We know that $P(E \cup F) = P(E) + P(F) - P(EF)$, so $r = p + q - P(EF)$. Rearranging, we get
   $$P(EF) = p + q - r.$$

   (b) (4 pts) Let H be the event that EITHER none of E, F OR both of E, F occur. In terms of p, q and r, write an expression for $P(H)$.
   Solution: H consists of $(E \cup F)^c$ (none of E, F occur) together with EF (both occur), and these are mutually exclusive, so their probabilities add. We have $P((E \cup F)^c) = 1 - P(E \cup F) = 1 - r$ and $P(EF) = p + q - r$, so
   $$P(H) = 1 + p + q - 2r.$$

2. (3 pts) 55% of students read the Observer daily, 25% live off campus, and 63% either live of campus or read the Observer daily (or both). I pick a student at random (all students equally likely). What is the probability that the student I pick BOTH lives off campus AND reads the Observer daily?

   Solution: Let S be the set of all students (the sample space for the experiment "pick a random student"). Let E be the event that a student reads the Observer and F the event that they live off campus. We are given that $P(E) = .55$, $P(F) = .25$ and $P(E \cup F) = .63$, and we want to compute $P(EF)$. Using the result of the first part of the quiz, with $p = .55$, $q = .25$ and $r = .63$, we have

   $$P(EF) = .55 + .25 - .63 = .17.$$