Math 30530, Fall 2009, homework 5 solutions

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- **2.** A and B are mutually exclusive; therefore, they are dependent. If A occurs, then the probability that B occurs is 0 and vice versa.

- **3.** Neither. Since the probability that a fighter plane returns from a mission without mishap is 49/50 *independent* of other missions, the probability that a pilot who flew 49 consecutive missions without mishap making another successful flight is still 49/50=0.98; neither higher nor lower than the probability of success in any other mission.

- **4.** $P(AB) = 1/12 = (1/2)(1/6)$; so A and B are independent.

- **5.** $(3/8)^3(5/8)^5 = 0.00503$.

- **11.** $(1 - 0.0001)^{64} = 0.9936$.

- **13.** **(a)** $P(A \cup B) \geq P(A) = 1$, so $P(A \cup B) = 1$. Now

$$1 = P(A \cup B) = P(A) + P(B) - P(AB) = 1 + P(B) - P(AB)$$

gives $P(B) = P(AB)$.

(b) If $P(A) = 0$, then $P(AB) = 0$; so $P(AB) = P(A)P(B)$ is valid. If $P(A) = 1$, by part (a), $P(AB) = P(B) = P(A)P(B)$.

- **17.** $1 - (0.3)(0.2)(0.1) = 0.994$.

- **23.** **(a)** $1 - \left[(n-1)/n\right]^n$. **(b)** As $n \to \infty$, this approaches $1 - (1/e) = 0.6321$.

- **29.** Let E_i be the event that the switch located at i is closed. The desired probability is

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- **1.** The set of possible values of X is $\{0, 1, 2, 3, 4, 5\}$. The probabilities associated with these values are

x	0	1	2	3	4	5
$P(X = x)$	6/36	10/36	8/36	6/36	4/36	2/36

• **3.** The set of possible values of X is $\{0, 1, 2 \ldots, N\}$. Assuming that people have the disease independent of each other,

$$P(X = i) = \begin{cases} (1-p)^{i-1}p & 1 \le i \le N \\ (1-p)^N & i = 0. \end{cases}$$

• **7.** Note that X is neither continuous nor discrete. The answers are

(a) $F(6-) = 1$ implies that $k(-36 + 72 - 3) = 1$; so $k = 1/33$.

(b) $F(4) - F(2) = 29/33 - 4/33 = 25/33$.

(c) $1 - F(3) = 1 - (24/33) = 9/33$.

(d) $$P(X \le 4 \mid X \ge 3) = \frac{F(4) - F(3-)}{1 - F(3-)} = \frac{\frac{29}{33} - \frac{9}{33}}{1 - \frac{9}{33}} = \frac{5}{6}.$$

• **5.** $P(X < 1) = F(1-) = 1/2$.

$P(X = 1) = F(1) - F(1-) = 1/6$.

$P(1 \le X < 2) = F(2-) - F(1-) = 1/4$.

$P(X > 1/2) = 1 - F(1/2) = 1 - 1/2 = 1/2$.

$P(X = 3/2) = 0$.

$P(1 < X \le 6) = F(6) - F(1) = 1 - 2/3 = 1/3$.

• **11.** F is a distribution function because $F(-\infty) = 0$, $F(\infty) = 1$, F is right continuous, and $F'(t) = \dfrac{1}{(1+t)^2} > 0$ implies that it is nondecreasing.