

Math 30530, Fall 2009, Homework 3 solutions

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- 3. $\binom{20}{6}\binom{25}{6} = 6,864,396,000.$
- 5. $\binom{N-1}{n-1} / \binom{N}{n} = \frac{n}{N}.$
- 9. $\binom{10}{5} / \binom{12}{7} = 0.318.$
- 11. The coefficient of $(2x)^3(-4y)^4$ in the expansion of $(2x - 4y)^7$ is $\binom{7}{4}$. Thus the coefficient of x^3y^2 in this expansion is $2^3(-4)^4\binom{7}{4} = 71,680.$
- 16. $\frac{\binom{50}{5}\binom{150}{45}}{\binom{200}{50}} = 0.00206.$
- 22. The answer is the solution of the equation $\binom{x}{3} = 20$. This equation is equivalent to $x(x-1)(x-2) = 120$ and its solution is $x = 6$.
- 29. We must have 8 steps. Since the distance from M to L is ten 5-centimeter intervals and the first step is made at M, there are 9 spots left at which the remaining 7 steps can be made. So the answer is $\binom{9}{7} = 36.$
- 41. The identity expresses that to choose r balls from n red and m blue balls, we must choose either r red balls, 0 blue balls or $r-1$ red balls, one blue ball or $r-2$ red balls, two blue balls or ... 0 red balls, r blue balls.

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- 1. (a) $\binom{2n}{n} \frac{1}{2^{2n}} = \frac{(2n)!}{n!n!} \frac{1}{2^{2n}} \sim \frac{\sqrt{4\pi n} (2n)^{2n} e^{-2n}}{(2\pi n) n^{2n} e^{-2n} 2^{2n}} \sim \frac{1}{\sqrt{\pi n}}.$
- (b) $\frac{[(2n)!]^3}{(4n)!(n!)^2} \sim \frac{[\sqrt{4\pi n} (2n)^{2n} e^{-2n}]^3}{\sqrt{8\pi n} (4n)^{4n} e^{-4n} (2\pi n) n^{2n} e^{-2n}} = \frac{\sqrt{2}}{4^n}.$

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• 5. $\frac{30 - 20}{30 - 15} = \frac{2}{3}$.

• 8. $4/30 = 0.133$.

• 9. $\frac{\binom{40}{2} \binom{65}{6}}{\binom{105}{8}} = 0.239$.

$1 - \sum_{i=0}^2 \frac{\binom{40}{8-i} \binom{65}{i}}{\binom{105}{8}}$

• 13. $P(A | B) = \frac{P(AB)}{b}$, where

$$P(AB) = P(A) + P(B) - P(A \cup B) \geq P(A) + P(B) - 1 = a + b - 1.$$

- 17. Reduce the sample space: There are 21 crayons of which three are red. Seven of these crayons are selected at random and given to Marty. What is the probability that three of them are red?

The answer is $\binom{18}{4} / \binom{21}{7} = 0.0263$.