Math 30530, fall 2009, Homework 1 Solutions

Page 9:

- **1.** For $1 \le i, j \le 3$, by (i, j) we mean that Vann's card number is i, and Paul's card number is j. Clearly, $A = \{(1,2), (1,3), (2,3)\}$ and $B = \{(2,1), (3,1), (3,2)\}$.

  (a) Since $A \cap B = \emptyset$, the events A and B are mutually exclusive.

  (b) None of $(1,1)$, $(2,2)$, $(3,3)$ belongs to $A \cup B$. Hence $A \cup B$ not being the sample space shows that A and B are not complements of one another.

- **4.** Denote the dictionaries by d_1, d_2; the third book by a. The answers are $\{d_1d_2a, d_1ad_2, d_2d_1a, d_2ad_1, ad_1d_2, ad_2d_1\}$ and $\{d_1d_2a, ad_1d_2\}$.

- **5.** EF: One 1 and one even.

  E^cF: One 1 and one odd.

  E^cF^c: Both even or both belong to $\{3, 5\}$.

- **7.** $S = \{x : 7 \le x \le 9\frac{1}{6}\}$; $\{x : 7 \le x \le 7\frac{1}{4}\} \cup \{x : 7\frac{3}{4} \le x \le 8\frac{1}{4}\} \cup \{x : 8\frac{3}{4} \le x \le 9\frac{1}{6}\}$.

- **13.** Parts (a) and (d) are obviously true; part (c) is true by DeMorgan's law; part (b) is false: throw a four-sided die; let $F = \{1, 2, 3\}$, $G = \{2, 3, 4\}$, $E = \{1, 4\}$.

Page 23:

- **3.** Let E be the event that an earthquake will damage the structure next year. Let H be the event that a hurricane will damage the structure next year. We are given that $P(E) = 0.015$, $P(H) = 0.025$, and $P(EH) = 0.0073$. Since

$$P(E \cup H) = P(E) + P(H) - P(EH) = 0.015 + 0.025 - 0.0073 = 0.0327,$$

  the probability that next year the structure will be damaged by an earthquake and/or a hurricane is 0.0327. The probability that it is not damaged by any of the two natural disasters is 0.9673.

- **5.** Let A be the event that a randomly selected investor invests in traditional annuities. Let B be the event that he or she invests in the stock market. Then $P(A) = 0.75$, $P(B) = 0.45$, and $P(A \cup B) = 0.85$. Since,

$$P(AB) = P(A) + P(B) - P(A \cup B) = 0.75 + 0.45 - 0.85 = 0.35,$$

  35% invest in both stock market and traditional annuities.

- **7.** In point of fact Rockford was right the first time. The reporter is assuming that both autopsies are performed by a given doctor. The probability that both autopsies are performed by the same doctor–whichever doctor it may be–is 1/2. Let AB represent the case in which Dr. A performs the first autopsy and Dr. B performs the second autopsy, with similar representations for other cases. Then the sample space is $S = \{AA, AB, BA, BB\}$. The event that both autopsies are performed by the same doctor is $\{AA, BB\}$. Clearly, the probability of this event is 2/4=1/2.

- **10.** $P(A \cup B) \leq 1$ implies that $P(A) + P(B) - P(AB) \leq 1$.

- **16.** 7/11.

- **18.** Let M and F denote the events that the randomly selected student earned an A on the midterm exam and an A on the final exam, respectively. Then

$$P(MF) = P(M) + P(F) - P(M \cup F),$$

where $P(M) = 17/33$, $P(F) = 14/33$, and by DeMorgan's law,

$$P(M \cup F) = 1 - P(M^c F^c) = 1 - \frac{11}{33} = \frac{22}{33}.$$

Therefore,

$$P(MF) = \frac{17}{33} + \frac{14}{33} - \frac{22}{33} = \frac{3}{11}.$$

- **22.** Let T and F be the events that the number selected is divisible by 3 and 5, respectively.

(a) The desired quantity is the probability of the event TF^c:

$$P(TF^c) = P(T) - P(TF) = \frac{333}{1000} - \frac{66}{1000} = \frac{267}{1000}.$$

(b) The desired quantity is the probability of the event $T^c F^c$:

$$P(T^c F^c) = 1 - P(T \cup F) = 1 - P(T) - P(F) + P(TF)$$
$$= 1 - \frac{333}{1000} - \frac{200}{1000} + \frac{66}{1000} = \frac{533}{1000}.$$