MATH 30440, SEC 01, SPRING 2010

HOMEWORK 5 SOLUTIONS

5.6) a)

$$\left\{\begin{array}{l} E(X) = 7 \Rightarrow np = 7 \\ Var(X) = 2.1 \Rightarrow np(1-p) = 2.1 \\ \text{Dividing}: \quad 1-p = \frac{2.1}{7} = .3 \\ \qquad p = .7 \\ \text{So } n = 10 \end{array}\right.$$

$$P(X=4) = \binom{10}{4} .7^4 .3^6 = .0368$$

b) $P(X > 12) = 0$

5.10) a) $P(X=2) = \binom{10}{2}(.1)^2(.9)^8 = .1937$

Poisson approximation: Use $\lambda = 10 \times .1 = 1$

$$P(X=2) = \frac{1^2}{2!} e^{-1} = .1839$$

b) .3487 (exact) versus .3679

c) .0660 (exact) versus .0723

5.11) Binomial trial, $n = 50$, $p = \frac{1}{100}$.

Use Poisson $\left(\frac{1}{2}\right)$ to estimate

a) $1 - P(X=0) = .3934\ldots$

b) $P(X=1) = \frac{\left(\frac{1}{2}\right)}{1!} e^{-\frac{1}{2}} = .3033$

c) $1 - P(X = 0 \text{ or } 1) = .0902$

5.12) $P(\text{No colds} \mid \text{drug effective}) = P(\text{Poisson}(2) = 0)$

$= .1353$

$P(\text{No Colds} \mid \text{drug not eff}) = P(\text{Poisson}(3) = 0)$

$= .0498$

$P(\text{drug effective}) = .75$

$P(\text{drug not eff}) = .25$

By Bayes,

$$P(\text{effective} \mid \text{No colds}) = \frac{.1353 \times .75}{.1353 \times .75 + .0498 \times .25}$$

$= .8908$

5.16) Exact probability:

$$\sum_{k=4}^{1000} \binom{1000}{k} (.001)^k (.999)^{1000-k}$$

Approximate: Use Poisson with $\lambda = 1$,

since $np = 1$

$$P(X>3) = .019 \quad \big(1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)\big)$$

5.18) $P(\text{kept}) = P(\text{Getting } 9 \text{ or } 10 \text{ in hypergeometric with } N = 80,\ M = 20,\ n = 10)$

$$= \frac{\binom{80}{9}\binom{20}{1}}{\binom{100}{10}} + \frac{\binom{80}{10}\binom{20}{0}}{\binom{100}{10}}$$

$$= .3630$$

5.20) a) $P(X=k) = (1-p)^{k-1} p$

b) $E(X) = \sum_{k=1}^{\infty} k(1-p)^{k-1} p$

$= p \sum_{k=1}^{\infty} k(1-p)^{k-1}$

$$\sum_{k=1}^{\infty} k x^{k-1} = \frac{d}{dx}\left(\sum_{k=1}^{\infty} x^k\right)$$

$$= \frac{d}{dx}\left(\frac{x}{1-x}\right)$$

$$= \frac{(1-x)+x}{(1-x)^2} = \frac{1}{(1-x)^2}$$

So $E(X) = p \times \frac{1}{(1-(1-p))^2} = \frac{1}{p}$

c) $P(\cancel{X}=k) = \binom{k-1}{r-1}(1-p)^{k-r} p^r$

locate $r-1$ successes among first $k-1$ trials

d) $Y = Y_1 + \dots + Y_r$ where

Y_i = # trials to go from success $i-1$ to success i

= Geometric (p)

$$E(Y) = E(Y_1) + \dots + E(Y_r)$$

$$= \frac{1}{p} + \dots + \frac{1}{p} = \frac{r}{p} \quad (\text{using } b))$$

5.21) U has range $(0,1)$ so

$V = a + (b-a)U$ has range a to $a + (b-a) = b$

Distribution of V:

$P(V \le a) = 0, \quad P(V \le b) = 1;$

$P(V \le t) = 0$ for any $t \le a$

$P(V \le t) = 1$ for $t \ge b$

For $a \le t \le b$:

$$P(V \le t) = P(a + (b-a)U \le t)$$

$$= P\left(U \le \frac{t-a}{b-a}\right)$$

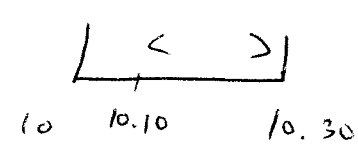
$$= \frac{t-a}{b-a}$$

So density of V is :

$$f(t) = \begin{cases} 0 & \text{if } t \le a,\ t \ge b \\ \frac{1}{b-a} & \text{if } a \le t \le b \end{cases}$$

This is exactly density of uniform on (a,b), so V is uniform on (a,b).

5.22) $P(\text{Wait} > 10) = \frac{2}{3}$



$P(\text{Wait} > 10 \text{ more minutes after } 10.15)$

$$= P(\text{Wait} > 25 \mid \text{Wait} > 15)$$

$$= \frac{P(\text{Wait} > 25)}{P(\text{Wait} > 15)} = \frac{1/6}{\frac{1}{2}} = \frac{1}{3}$$