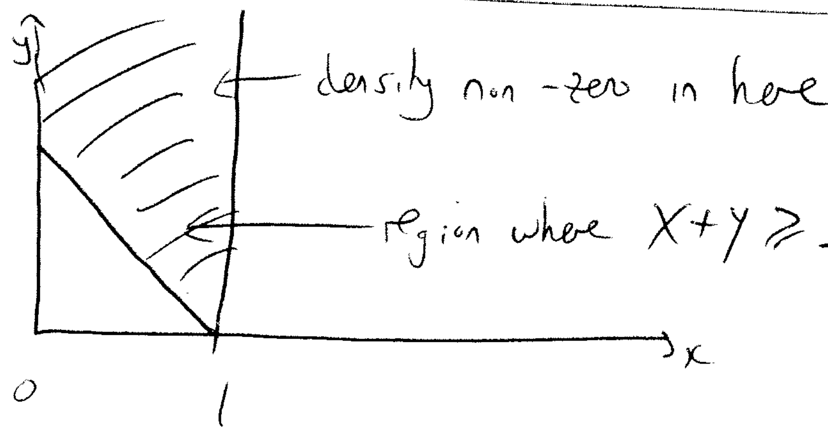
MATH 30440, SEC 01, SPRING 10

EXAM 2 PRACTICE PROBLEMS - SOLUTIONS

1)



$$f(x,y) = \begin{cases} e^{-y} & 0 \leq x \leq 1,\ 0 \leq y \leq \infty \\ 0 & \text{otherwise} \end{cases}$$

$$P(X+Y \geq 1) = \iint_{\text{shaded}} e^{-y}\, dA$$

$$= 1 - \iint_{\Delta} e^{-y}\, dy$$

$$= 1 - \int_0^1 \int_0^{1-x} e^{-y}\, dy\, dx$$

$$= 1 - \int_0^1 \left[-e^{-y}\right]_0^{1-x} dx$$

$$= 1 - \int_0^1 (1 - e^{x-1})\, dx$$

$$= 1 - \left[x - e^{x-1}\right]_0^1 = 1 - \frac{1}{e}$$

2) a) Binomial $n = 53$

$p = .999$ (success = not defective)

From binomial calculator,

$P(\geq 50) = 1.0000$

b) From binomial calculator,

$P(\geq 50 \text{ if buys } 51) = .9988$

$P(\geq 50 \text{ if buys } 50) = .9512$

So <u>51</u> needed

3) X = amount of fill

$= \text{Normal}(\mu, (.3)^2)$

$P(\text{overflow}) = .01$ same as

$P(X > 8) = .01$

$P\left(Z > \frac{8-\mu}{.3}\right) = .01$

Since $P(Z > +2.33) = .01$,

same as $\frac{8-\mu}{.3} = 2.33$, $\mu = 7.301$.

4) R = resistance = Normal (μ, σ^2)

$P(R > 10.257) = .0985$ same as

$P\left(Z \geq \frac{10.257 - \mu}{\sigma}\right) = .0985$ or

$$\frac{10.257 - \mu}{\sigma} = 1.29 \quad ①$$

$P(R < 9.671) = .0505$ same as

$P\left(Z \leq \frac{9.671 - \mu}{\sigma}\right) = .0505$ or

$$\frac{9.671 - \mu}{\sigma} = -1.64 \quad ②$$

Solving ① and ② get $\sigma = .2$
$\mu = 10$

5) $E(2^X) = \sum_{k=0}^{\infty} 2^k \frac{2^k}{k!} e^{-2}$

$= e^{-2} \sum_{k=0}^{\infty} \frac{4^k}{k!}$

$= e^{-2} e^{4} = e^{2}$.

6) Density of X is $f(x) = \begin{cases} 0 & x<0,\ x>2 \\ \frac{1}{2} & 0 \le x \le 2 \end{cases}$

$$E(X^3 - X) = \int_0^2 (x^3 - x)\tfrac{1}{2}\,dx$$

$$= \left[\frac{x^4}{8} - \frac{x^2}{4}\right]_0^2$$

$$= 1$$

7) Use exponential rv to model arrival time of first car.

λ_1 = average # arrivals per hour

$= 10$

Use Poisson to model # arrivals in 30 minutes

λ_2 = average # arrivals per $\frac{1}{2}$ hour

$= 5$

a) $P(\le 1$ arrival in $\frac{1}{2}$ hour$)$

$= P(\text{Poisson}(5) \le 1) = e^{-5} + 5e^{-5}$

$= .04\ldots$

b) P(first car arrives within 10 minutes)

$= P(\text{exponential}(10) \le \frac{1}{6})$

$= \int_0^{\frac{1}{6}} 10e^{-10x}\,dx = \left[-e^{-10x}\right]_0^{\frac{1}{6}}$

$= 1 - e^{\frac{-10}{6}}$

$= .81\ldots$

8) # disintegrating in 1 millisecond is Binomial $(10^{16}, 10^{-15})$.

Use Poisson (10) to approximate it

P(no more than 2) $= P(\text{Poisson}(10) \le 2)$

$= e^{-10} + 10e^{-10} + \frac{100}{2}e^{-10}$

$= .0023\ldots$

9) Gordon's original weight:

$$210 + \text{Normal}\left(.5, (.8)^2\right) \text{ error}$$

$$= \text{Normal}\ (219.5, .64)$$

Gordon's new weight $= \text{Normal}\ (201.5, .64)$

Gordon has lost

$$\text{Normal}\ (219.5, .64) - \text{Normal}\ (201.5, .64)$$

$$= \text{Normal}\ (8, 1.28)$$

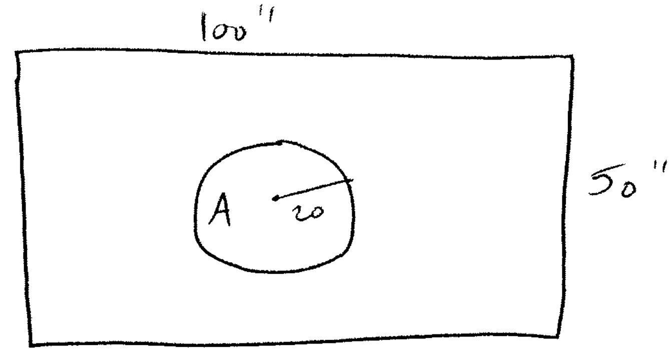
Similarly, Kyle has lost Normal $(6, .72)$

P(Gordon's loss greater) =

$$P\left(\text{Normal}\ (8, 1.28) > \text{Normal}\ (6, .72)\right)$$

$$= P\left(\text{Normal}\ (2, 2) > 0\right)$$

$$= P\left(Z > \frac{-2}{\sqrt{2}}\right) = .9207$$

10) 

Want $P(\text{Ball ends up in } A) = \dfrac{\text{Area of } A}{\text{Area of table}}$

$$= \frac{400\pi}{5000} = \frac{\pi}{25}$$

11) X = # house fires

Model X as Poisson, $\lambda = 50$

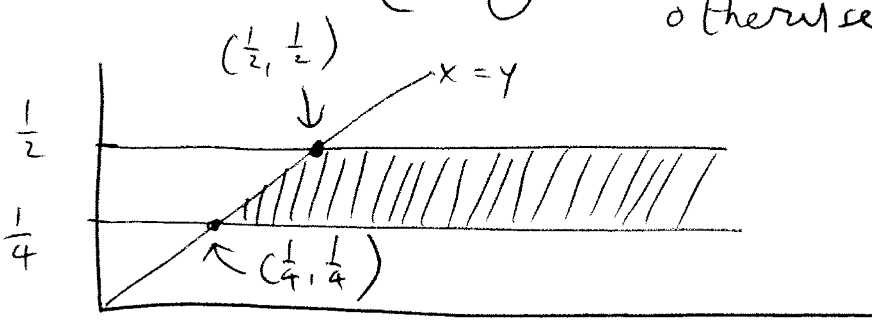
$$P(X = 15) = \frac{50^{15}}{15!} e^{-50}$$

12) X = time to spraying = exponential (1)

Y = time to toy = Uniform $(\frac{1}{4}, \frac{1}{2})$

Joint density of X and Y:

$$f(x,y) = \begin{cases} e^{-x} \dfrac{1}{\frac{1}{4}} & 0 \le x < \infty,\ \frac{1}{4} \le y \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$



Want $P(Y < X)$

$$P(Y < X) = \iint f(x,y)\,dA$$

$$= \int_{\frac{1}{4}}^{\frac{1}{2}} \int_{\frac{1}{4}}^{x} 4e^{-x}\,dy\,dx$$

$$+ \int_{\frac{1}{2}}^{\infty} \int_{\frac{1}{4}}^{\frac{1}{2}} 4e^{-x}\,dy\,dx.$$

13) X = lifetime of belt

$= \text{exponential}\left(\frac{1}{30\,000}\right)$

$E(X) = 30{,}000$

By memorylessness, this is the expected further lifetime, given it's still working after 25,000 miles.

14) $\bar{X}$ = average of 30 rvs, $\mu = 80$
$\sigma^2 = 120$

$\bar{X}$ has mean 80, variance $\frac{120}{30} = 4$,
and (by CLT) is approximately normal.

$$P(\bar{X} > 85) = P\left(Z > \frac{85-80}{\sqrt{4}}\right)$$
$$= P(Z > 2.5)$$
$$= .62\%$$