Math 30440 — Probability and Statistics

Spring 2010 first mid-term exam practice problems

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1. A random variable X is given by density function

$$f(x) = \begin{cases} \frac{c}{x^4} & \text{if } x \geq 4 \\ 0 & \text{if } x < 4 \end{cases}$$

   (a) Find c.

   (b) Calculate the expectation and variance of X.

   (c) Find a number t such that $P(X \geq t) = 1/2$.

   (d) Calculate $E(X^2 + X)$.

2. A chemical reaction takes place it two steps. The time it takes for the first step to complete is a random variable X given by density function

$$f(x) = \begin{cases} 2e^{-2x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$$

   The second step begins as soon as the first has finished, and the time it takes for the second step to complete is a random variable Y given by density function

$$f(y) = \begin{cases} 3e^{-3y} & \text{if } y \geq 0 \\ 0 & \text{if } y < 0. \end{cases}$$

   (a) What is the probability that it takes at most 4 time units for the whole reaction to take place? (That is, what is $P(X + Y \geq 4)$?)

   (b) Write down (no need to evaluate) an integral whose value is $E(X + Y)$.

3. If four different fair dice (each of the numbers 1 through 6 equally likely) are tossed, what is the probability that they will show four different numbers?

4. Professor Bunsen always starts his Alchemy 231 lecture course with one of the three great alchemical experiments: turning lead into gold (20% of all times that he teaches the course), brewing the elixir of life (40% of the times) and creating the Philosopher's stone (40% of the time). When he tries to turn lead into gold,

the result always ends with a explosion; when he brews the elixir of life, there is a 50% chance of an explosion, and when he creates the Philosopher's stone, 8 times out of 10 there is an explosion. The Dean wants to see which experiment Professor Bunsen will do this year, but he arrives late. If he see the lecture-hall filled with post-explosion smoke, what should he conclude is the probability that he has just missed a demonstration of brewing the elixir of life?

5. Experience has shown that the number of people who enter a certain office building per minute during the middle of a quiet day is a random variable X with mass function
$$p(x) = \frac{1}{3}c^x, x = 0, 1, 2, 3, \ldots.$$
   (a) What is c?
   (b) What is the probability that between 2 and 5 people (inclusive) enter the building in a given minute?

6. Let A and B be events with $P(A) = x$, $P(B) = y$ and $P(AB) = z$.
   (a) Write an expression for the probability that A occurs but not B, in terms of x, y and z.
   (b) Write an expression for the probability that neither A nor B occurs, in terms of x, y and z.

7. A bowl contains 20 chips of which 9 are red, 8 are white, 3 are blue. We pick 6 chips from the bowl. Find the probability that
   (a) each of the 6 chips is red
   (b) we extracted 3 red, 2 white and 1 blue chip
   (c) we picked at least 1 white and at least 1 blue chip

8. I toss a fair coin (one that is equally likely to come up Heads as Tails) 3 times, and let X be the number of Heads minus the number of Tails.
   (a) Calculate the probability mass function of X as well as $E(X)$ and $\mathrm{Var}(X)$.
   (b) Repeat for a coin that has probability p of coming up Heads.

9. A gambler plays a game for which he knows he will win \$10 with probability 1/2, \$20 with probability 1/3 or \$30 with probability 1/6. What is the expected gain and what is the variance of the gain?

10. The IQ of a randomly chosen student on campus is a random variable with mean 120 and standard deviation 8. Use Tcebychev's inequality to find a number t such that the probability that a randomly chosen student's IQ is within t of 120 (either above or below) is at least 90%.