Introduction to Operations Research (Math 30210) Fall 2014

Worksheet on Solver

September 18 2014

Introduction

Solver is an add-in to Microsoft Excel that is capable of solving a wide variety of optimization problems (problems of the form "maximize/minimize an objective function of some variables, subject to some constraints on the variables"). In particular, it is quite good at solving linear programming problems. This worksheet introduces you to Solver as a tool for resolving linear programming problems.

1. Section 1 describes how to load Solver on to Excel.

2. Section 2 describes how to enter the data of a basic LP problem into Excel.

3. Section 3 and 4 describe how to prepare Solver to solve the problem.

4. Section 5 describes how to solve the problem using Solver.

5. Section 6 briefly mentions some other useful features of Solver.

6. Section 7 gives a more involved example of entering data in Excel.

7. Section 8 gives some exercises to work through.

You can follow along with the main example (Sections 2 through 5 via screenshots in this document, but also by looking at the Excel file `mixture_example.xlsx` on the course website. For the more involved example, use the file `transport_example.xlsx`.

For more information on Solver and LP problems, look at Appendix E and Section 3.10 of the text. For technical problems, you can talk to the nice people at the OIT help desk (128 DeBartolo). I'll return from my trip on Tuesday, September 23; you can also bring up any issues with me that day.

The exercises associated with this worksheet are due Friday September 26.

1 Loading Solver

Solver is an "Add-in", meaning that it has to be loaded in to your copy of Excel before it can be used. OIT has told me that all machines in campus labs that run Excel already have Solver added in, but if you are using a personal machine you may need to manually load it.

What I'm about to describe is valid for Excel 2013 for Windows. It's essentially the same for other/earlier versions of Solver, but there may be minor cosmetic differences. To get specific information about loading Solver on a Mac, go to `http://support.microsoft.com/kb/2431349`.

To see if your version of Excel already has Solver, go to the DATA tab, and see if "Solver" appears as one of the options. If not, you'll need to take the following steps:

1. Under FILE select "Options", then "Add-Ins".

2. Select "Excel Add-ins" from the "Manage" drop-down menu (it should be the default), and click on the "Go" button.

3. Check "Solver Add-in", and then click on the "ok" button.

Solver should now be accessible under the DATA tab.

2 Entering the data for an LP problem

There is no right or wrong way to enter the data for an LP problem into Excel; no matter how you have entered the data, you'll be able to specify the precise problem you are working with once you launch the Solver Add-in. However, some ways of organizing data are better than others, both from the point of view of being able to keep track visually of what's going on, and from the perspective of specifying in Solver what the constraints are.

Here are the steps I might take to organize my data, for a basic problem. You may prefer a slightly different paradigm; that's ok! To illustrate my process, I'll use question 2 from homework 1 (asking what is the lowest-cost mix of three liquids, each with certain percentages of antifreeze and additives, with the mix required to have at least a certain percentage of antifreeze and a certain percentage of additives; I won't write the exact problem here, but you can look it up in the textbook on page 20 (problem 14)). You can see the finished result of this process in the file `mixture_example.xlsx` on the website.

1. On a piece of paper, write down all the variables (decision variables, auxiliary variables, slack variables, etc), what they represent, and a short, helpful name for each one. Example:

$$x = \text{amount of A used per gallon} \quad [\text{Liq. A}] \tag{1}$$
$$y = \text{amount of B used per gallon} \quad [\text{Liq. B}] \tag{2}$$
$$z = \text{amount of C used per gallon} \quad [\text{Liq. C}]. \tag{3}$$

2. Next, write the objective function, giving it a short, helpful name, and saying whether it is to be minimize or maximized. One after another below that write all of the major

constraints that the variables must satisfy, with the variables on the left-hand side, then a $\leq$, or $=$, or $\geq$, then the constant, then a short, helpful name for the constraint. As much as possible keep the variables aligned so that all occurrences of x, for example, come in a single column. "Major" constraints here means all constraints other than non-negativity and integrality constraints. On a final line, say which variables are constrained to be non-negative, which are unconstrained, and which are integral. Example:

$$
\begin{array}{rlrrrrrl}
\text{Minimize} & \text{cost/gallon} & = & 1.6x & +.5y & +1.4z & & \\
\text{subject to} & & & 60x & +18y & +75z & \geq & 50 & [\text{antifreeze}] \\
& & & 10x & +3y & & \geq & 5 & [\text{additivies}] \\
& & & x & +y & +z & = & 1 & [\text{gallon}] \\
& & & x \geq 0 & y \geq 0 & z \geq 0. & & &
\end{array}
$$

Notice that there is no need to convert the problem to standard form.

3. Now transfer this data, together with the short, helpful names, into Excel, more-or-less in the same format, just without using the x, y, z's (i.e., just recording coefficients). Rather than labouriously explaining how I do this, I'll just give a screenshot (fig 1).
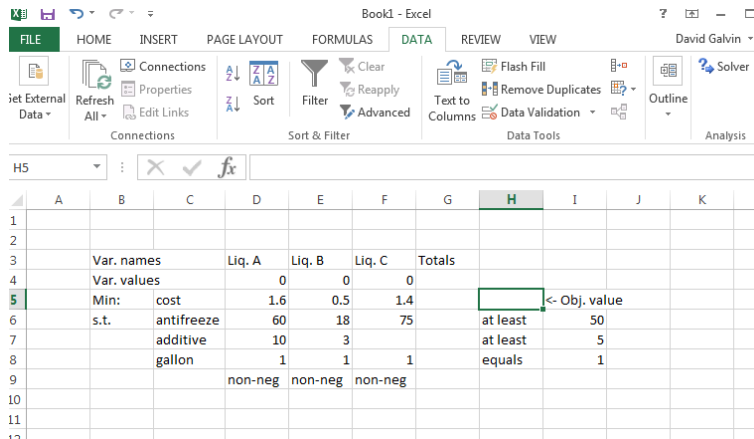

Figure 1: One way to enter the data for an LP problem into Excel in an organized way

A few points to note:

- Cells D4, E4, F4 are where the values of the variables x, y, z at optimum will appear. I have entered "0" into each cell, but there isn't any need to enter anything.

- Cell G5 is where the value of the objective at optimum will appear.

- Notice one extra column that wasn't in my handwritten representation: the "Totals" column G. This column is going to be used to express the value that the left-hand side of each constraint takes, for a given value of the variables. It's need for the technical reason that Solver prefers to see constraints presented to it in the form of the information "what's in this one cell should be at least what's in this other cell", rather than "a certain function of what's in these cells should be

at least what's in this other cell", so we have to do the computing of the left-hand side of each constraint in Excel rather than in Solver.

- Everything I've entered so far is raw data; I've not specified any equations/inequalities yet.

3 Preparing the data for Solver

Before Solver can do its job, a little preparatory work is needed on the data.

1. Tell Excel that the objective value, appearing in cell H5, is calculated from the variable values and the coefficients appearing in the objective row. Do this by clicking in cell H5, and then in the equation box typing "=$D$4*$D$5+$E$4*$E$5*$F$4*$F$5". But note that there is another, cleaner way to express this sum-of-products, and that's using Excel's SUMPRODUCT command. In this case, you would type into the equation box "=SUMPRODUCT($D$4:$F$4,$D$5:$F$5)" (this tells Excel to multiply each entry in the string of cells from D4 to F4, by the corresponding entry in the string from D5 to F5, and then add the results). Notice that "0" now appears in cell H5; that's the value of the objective when all variables are set to 0, in this case. See screenshot (fig 2).
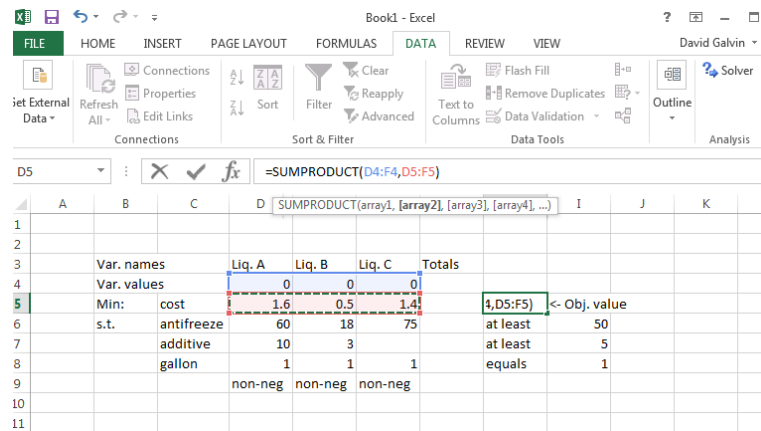

Figure 2: Telling Excel what the objective function is

2. Next, tell Excel what the left-hand side of the each constraint evaluates to, as a function of the variable values. For the antifreeze constraint, for example, we could enter into cell G6 either "=$D$4*$D$6+$E$4*$E$6*$F$4*$F$6", or the simpler-to-parse "=SUMPRODUCT($D$4:$F$4,$D$6:$F$6)"; this tells Excel to enter into cell G6 the value that the left-hand side of the antifreeze constraint takes, when the variable values are as given by the entries in cells D4 through F4. Do the same (in cells G7, G8) for the remaining constraints.

4 Setting up Solver

Now it is time to launch Solver, by going to the DATA tab and clicking on "Solver". You'll get a dialog box like the one you see in screenshot (fig 3). A number of steps need to be taken to tell Solver exactly what problem you are trying to solve.
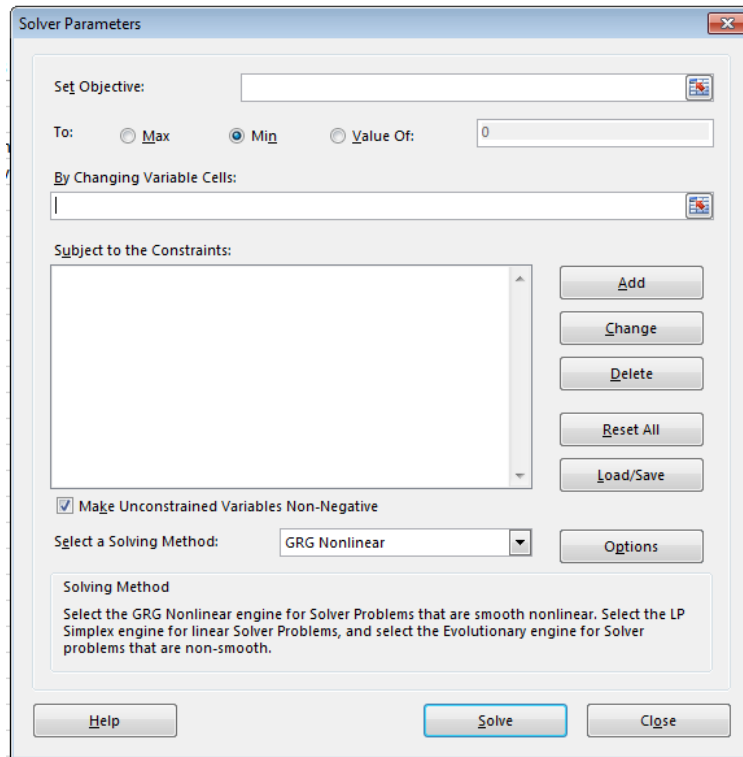

Figure 3: The Solver dialog box

1. Make sure that the cell named in the "Set Objective" box is the cell you have reserved for the value of the objective function. In our example, you would achieve this by clicking in the "Set Objective" box, clearing whatever text is there, and then either entering "$H$5" or clicking on cell H5. Next, click on the radio button "Min" (because the problem is a minimization). Next, in the "By Changing Variable Cells" box, enter the cell numbers that have been reserved for the variable values; in our example we would enter "$D$4:$F$4" (all cells between D4 and F4 inclusive). Solver now knows that its task is the minimize the function of these variables that has been keyed into cell H5.

2. Next, enter the constraints of the problem. Click on "Add", and a new dialog box pops up. See screenshot (fig 4). For the first constraint, in the "Cell Reference" box enter the name of the cell that we have used to calculate the value of the left-hand side of the constraint; for us, this is $G$6 (as before, you can either type "$G$6" into the "Cell Reference" box, or click on cell G6 while the cursor is in the "Cell Reference" box). In the middle box select "≤" if the constraint is a less-than-or-equal-to constraint, "≥"

if it is a greater-than-or-equal-to constraint (as it is in our case), and "=" if it is an equality; then in the "Constraint" box enter the cell corresponding to the right-hand side of the constraint (in our case, "$I$6"). The screenshot (fig 5) shows all this.
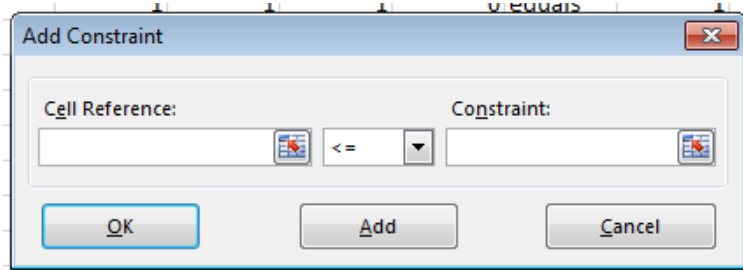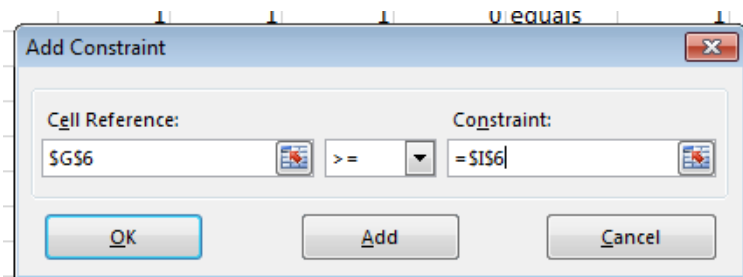

Figure 4: The constraint entry box

Figure 5: A fully recorded constrained

3. Now click on "Add" to record the constraint; you should be brought back to the main Solver dialog box, and it should look like the screenshot (fig 6).

4. Repeat this process for the other two constraints.

5. Now we come to the non-negativity constraints. Since *all* the variables in this problem are non-negative, you can simply check the box "Make Unconstrained Variables Non-Negative" (this will probably be checked by default). If some variables were constrained to be non-negative, and some were unconstrained, you would have to *uncheck* this box, and for each non-negative variable enter as a constraint that it is at least 0 (for x in this problem, for example, you would enter the constraint "$\$D\$4 \geq 0$" by putting "$\$D\$4$" in the "Cell Reference" box, and manually entering "0" in the "Constraint" box of the "Add" dialog box).

6. Finally, we need to tell Solver *how* it is to solve the LP. By far the best method that Solver has available to it for this kind of problem is the Simplex method, so on the drop-down menu "Select a Solving Method" select "Simplex LP". The Solver dialog box should now loom like the screenshot (fig 7).
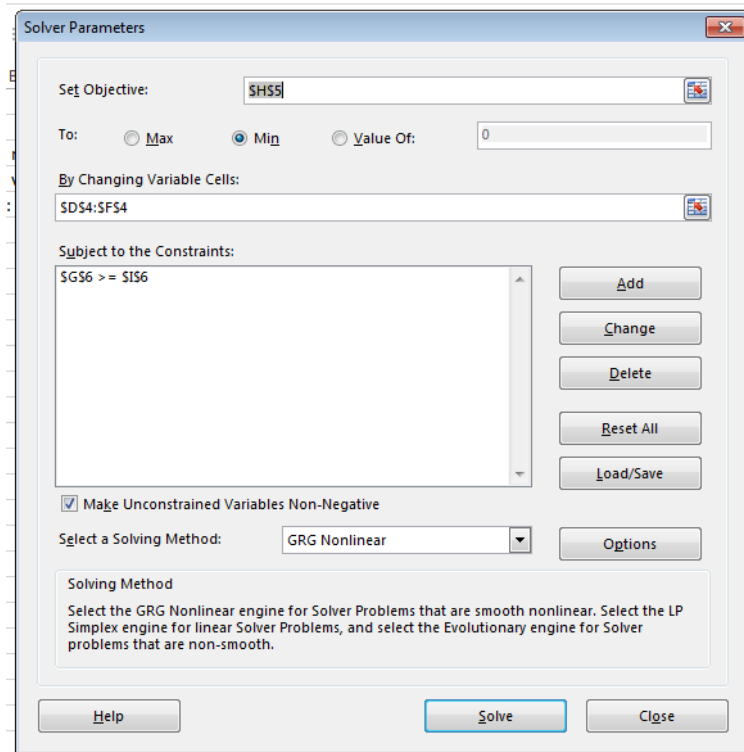
Figure 6: The Solver dialog box after a constraint has been added

5 Solving the LP

This is as simple as clicking "Solve" on the Solver dialog box. Solver will pop up a dialog box announcing the results. If a solution is found, then the value of the variables at the optimum are populated into the variable boxes on the Excel spreadsheet, and so of course the value of the objective function at the optimum, as well as the value of the left-hand side of all of the constraints, are also populated. This allows you to read off the optimum, how it achieved, and which constraints are satisfied tightly at the optimum, and which leave some slack. See the screenshot (fig 8) for all of this. For the running example, we learn that an optimal solution mixes .4 gallon of liquid A with .33 gallon of B with .26 gallons of C to make a mixture that has exactly 50% antifreeze and 5% additives, and costs $1.18 per gallon.

One very nice thing about Solver is that now that we have done all this work setting up the problem, we can solve modified versions of it very easily. For example, suppose we learn that the final mixture must have at least 55% antifreeze. Rather than solving a whole new problem, we can simply return to the Excel spreadsheet and change the "50" in cell I6 to "55". We then click on "Solver" again, where the entire set-up of the problem is already coded in. Clicking on "Solve" now quickly gives the solution to the new problem.

Another nice thing about Solver is that it is possible to get reports on the problem that contain a wealth of information. For example, the "Sensitivity report" allows you to quickly determine how much one can change the data of the initial problem without changing the optimal solution. We may discuss this more, alter in the semester. More relevant to a basic use of Solver is the "Answer report". In the "Reports" box of the Solver solution dialog box
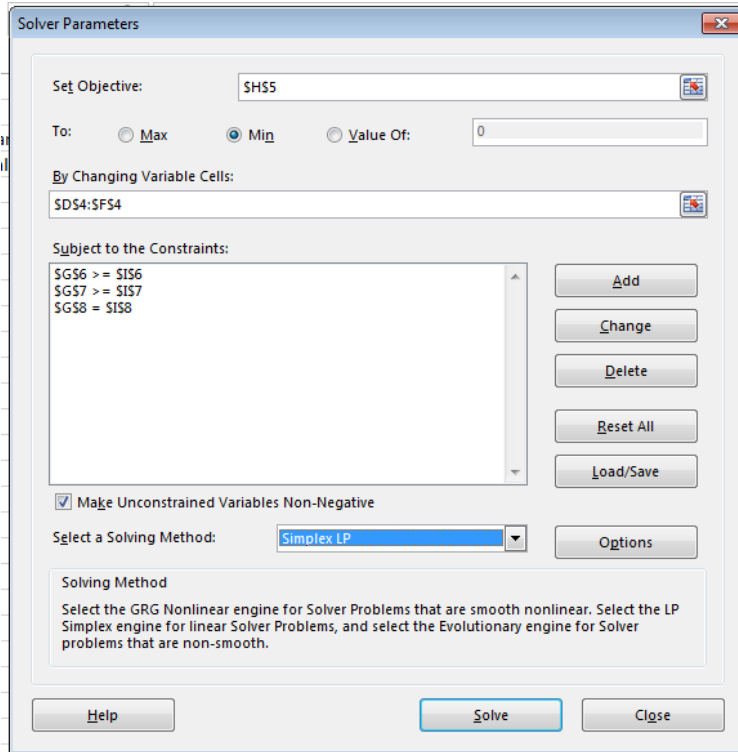
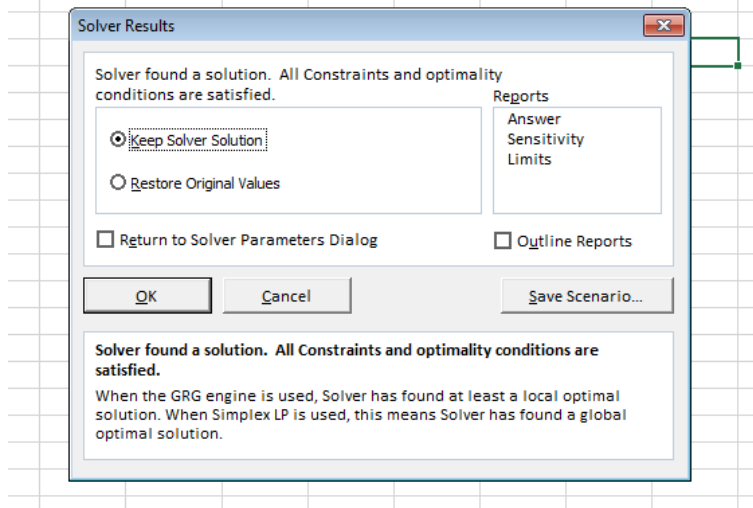Figure 7: The Solver dialog box just before solving the LP

Figure 8: The Solver dialog box, and the Excel spreadsheet, just after solving the LP

that pops up when you hit on "Solve" in the main Solver dialog box, click on "Answer" and then hit "Ok". A new sheet will be created in Excel, that contains the basic information about what Solver did: the solution it reached (both objective value and variable values at optimum), and how it took to reach the solution, among other things. See the screenshot (fig 9) for a the Answer report for the example problem.

Figure 9: Part of the Answer report

6   Some other features of Solver

Here I'll mention two other features of Solver that will be immediately useful if you are using it to check your answers to homework problems.

1. In the dialog box that allows you to enter the details of a constraints, we've seen using the drop-down menu in the middle box to specify that the value in one cell should be greater than or equal to, less than or equal to, or just plain equal to, the value in another cell. There is also the option "int" in that dialog box. Use that option to indicate that the value of a particular variable (whichever one whose location you have named in the "Cell Reference" box), ought to be an integer. In this case, you would leave the "Constraint" box empty.

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2. There is also the option "bin". Use this in the same way as "int" to indicate that the value of a particular variable must be either 0 or 1. This function saves you from having to use two constraints, one indicate that the value of the variable is an integer, and the other indicating that it is at most 1.

3. Once you have entered and solved a problem, you can easily change the parameters of the problem, and explore how that changes the solution: if, for example, the availability of some good changes from 100 units to 150 (or, you want to explore the effect the change would have), simply go to the cell in the Excel spreadsheet that has the "100" (recording the current availability of the good), and change to "150"; then re-open the Solver dialog box (which will continue to have all constraints recorded), and hit "Solve". You will get the new solution, which you can compare to the old one. For this to work well, it's important that every constraint is presented in terms of cell references rather than absolute numbers, so that the change from 100 to 150 is reflected in the entire problem, just by changing the value in that one cell. (See the second bullet point in Section 7 for another illustration of this point.)

4. It is also easy to add/remove constraints from the problem. Adding a new constraints has already been described. To remove an existing constraint, go to the Solver dialog box, click on that constraint, and then click "Delete". To see the effect that removing that constraint has on the problem, click "Solve" to get the new solution, and compare with the old.

Solver has many other features, which we may discuss later in the semester.

7 Another example

The previous example was quite straightforward, and so it was quite reasonable to encode all of the data and constraints in a single, compact array, with sparse notation. Larger examples, that might requiring calling on the variables many times in many different ways, both while calculating constraints and calculating the objective, need to be handled in a more expansive way in Excel. To illustrate what I mean, I will do another example. Rather than talking through the creation of it, I will simply direct you to the Excel file `transport_example.xlsx` on the website, where you can see all my data entry, my completed Solver dialog box, and my Answer report.

The example I'm using is the transportation problem Problem set 2.4, question 4 (page 37 of the textbook); I won't restate the problem here. Here are a few things to notice in the Excel file:

- For some parts of the constraint entry, I didn't try to go for the jugular and enter everything at once. For example, I created separate cells for the shipping costs from each of the three warehouses, and separate cells for the storage costs at each of the three warehouses. This might be useful information to know, but also it makes it easier to parse the final objective.

- Where possible, every constraint is presented in terms of cell references rather than absolute numbers (for example, the second labor constraint puts "$G9/2$" rather than "120/2" on the right-hand side); this also the raw numbers to be modified relatively easily, without having to worry about modifying the formulae for the constraints and the objective.

- There's no right or wrong way to enter data; just do what a) feels right to you and b) you feel could fairly easily be followed by someone else.

8 Exercises

1. Solve Problem set 2.3, question 15 (page 30 of the textbook, from the first homework), using Solver. Use the decision variables that I described in my solutions to the homework problems posted at `http://www3.nd.edu/~dgalvin1/30210/30210_F14/h1sols.PDF` (it's problem 4 there). Don't forget that it is a problem with integer constraints! After getting the solution, generate the answer report (it will appear as a new sheet inside your Excel file). Name your Excel file `florist_YOURNAME`.

2. Open the file `transport_example.xlsx` from the course website. By suitably modifying the Solver dialog box, answer each of the following questions. Record your answers in the space provided in the spreadsheet. Save the Excel file as `transport_YOURNAME`.

   (a) If labour constraint 1 is removed (but 2 kept), does the optimum cost change?

   (b) If labour constraint 2 is removed (but 1 kept), does the optimum cost change?

   (c) With both labour constraints restored, if storage costs at Wareh. 1 drop to 0, and it becomes possible to sell surplus units at Wareh. 2 for $8 per unit (with a guarantee that all surplus units will be sold), what does the optimum cost become?

   (d) With the storage cost changes as described above, what happens to the optimum cost if demand at outlet 1 jumps up to 150 units, and at outlet 2 drops down to 100 units?

3. Appendix B (page 421 of the textbook) gives an example of a simple LP problem where some unfortunate choices of places to pivot leads to cycling. Enter the problem as stated into Solver, and try to solve it. Generate an answer report. Name your Excel file `cycle_YOURNAME`.

4. Email me each of `florist_YOURNAME`, `transport_YOURNAME`, `cycle_YOURNAME`, by Friday, September 25.